FUZZY TIME SERIES FORECASTING MODEL USING COMBINED MODELS

V.Vamitha*

Abstract

Since nineteenth century, many researchers proposed various methods for forecasting enrollments, temperature prediction, stock price etc. In this paper a new model is introduced to forecast the fuzzy time series. By using Markov chain model the adjusted forecasted values are obtained and with aggregated fuzzy relationship the forecasted values are obtained. Using these two values, the proposed method is used to improve the accuracy. By giving example, forecasting the fuzzy time series are explained. The University of Alabama is used for illustration. Finally error analysis is made and error percentage is calculated and compared with existing methods. The proposed model confirms the potential benefits of the proposed approach with very small AFER.

Keywords:
Fuzzy time series model; Markov chain; fuzzy logic group; latest information; over all information.

Author correspondence:

V.Vamitha
Department of mathematics, B.C.M.W.G.Polytechnic college, Ettayapuram
Email: vvamitha80@gmail.com

1. Introduction

Markov chains are useful tools in modeling many practical systems such as queuing systems, manufacturing systems and inventory systems. Song and Chissom [19] proposed fuzzy time series models and fuzzy forecasting to model and forecast process whose observations are linguistic values. They illustrate the methodology by forecasting the enrollment at the University of Alabama from 20 years of data. Song and Chissom [19] used time-invariant fuzzy time series model, while Song and Chissom [20] used a time-variant model for the same problem. In their example the crisp data were fuzzified into linguistic values to illustrate the fuzzy time series method using fuzzy set theory. They asserted that all traditional forecasting methods fail when the enrollment data are composed of linguistic values. However a Markov model, described by Sullivan and Woodall [21] can use linguistic labels directly, but with the membership functions of the fuzzy approach replaced by analogous probability functions. Traditionally, the parameters of Markov model are estimated from observations in which the state occupied is known with certainty. Both the fuzzy forecasting and Markov models use the linguistic values directly to produce a ‘fuzzy’ forecast that, in turn, is ‘defuzzified’ into a numerical point estimate. A discrete-state Markov process has a (possibly infinite) number of states or categories that are mutually exclusive. The Markov property requires that the probability of transition to a particular state j, given the process is currently in state i, be independent of the history of states occupied before the current state. Typically with a finite number of states these transition probabilities are arrayed in a p × p transition matrix, where p is the number of states in the model. The element in row i and column j gives the probability of transition to state j given the current state is i. If we let , denote the

* Doctorate Program, Linguistics Program Studies, Udayana University Denpasar, Bali-Indonesia
vector of state probabilities at time t, where entry n in this vector gives the probability that the system is in state n at t, then

\[ P_{t+1} = P_t \times R_m , \]

where \( R_m \) is the transition matrix. The product \( P_t \times R_m \) gives the vector of state probabilities for time \( t + k \). Thus, in general,

\[ P_{t+k} = P_t \times R_m^k , \quad k = 1, 2, \ldots \]

The transition matrix \( R_m \) may vary with time, in which case a subscript is added to indicate the time to which it applies. Even though the state of a Markov chain is mutually exclusive, the process being modeled does not have to occupy one particular state with certainty for a Markov model to be valid. Several states can have non-zero probabilities, analogous to the concept of fuzzy set membership. With the Markov model, however, there is the requirement that all the state probabilities must sum to one for each observation. Membership functions do not have this restriction. From the experimental result of fuzzy time series available in the literature, the forecasting accuracy is based on effective length of intervals, fuzzy logical relationships and datum considered. We observe the forecasting accuracy is impacted not only by the fuzzy logical relationships but also by the latest historical data. Therefore, it may reduce the forecasting accuracy when the variation of latest time cannot be accounted into forecasting factors. To reconcile this problem, first a new method which aggregates overall information of fuzzy logical relationships with latest information should be developed to find forecasting values.

The experimental results show that the proposed model has proved an effective tool in the prediction of enrolment. The paper is organised as follows. Section 2 introduces the concept of the fuzzy time series model. Section 3 proposes a fuzzy time-series-Markov chain model and aggregated fuzzy relationship. Section 4 presents an illustration forecasting the enrolment at the university of Alabama and comparison of error with the existing results. Section 5 summarises the conclusion.

2. Basic concept of fuzzy time series.

Definition 1: Let \( Y(t) (t = 0,1,2,3,\ldots) \) be a subset of \( R \), be the universe of discourse on which fuzzy sets \( f_i(t) \) \((i = 1,2,3,\ldots)\) are defined and let \( F(t) \) be the collection of \( f_i(t). \) Then, \( F(t) \) is called as fuzzy time series on \( Y(t)(t = 0,1,2,3,\ldots). \) From this definition, we can see that

1. \( F(t) \) is a function of time
2. \( F(t) \) can be regarded as a linguistic variable, which is a variable whose values are linguistic values represented by fuzzy sets.
3. \( f_i(t) (i = 1,2,3,\ldots) \) are possible linguistic values of \( F(t), \) where \( f_i(t) (i = 1,2,3,\ldots) \) are represented by fuzzy sets.

Definition 2: Suppose \( F(t) \) is caused only by \( F(t-1) \) and is denoted by \( F(t-1) \rightarrow F(t) \), then there is a fuzzy relationship between \( F(t) \) and \( F(t-1) \) and can be expressed as the fuzzy relational equation \( F(t) = F(t-1) \circ R(t, t-1) \). Here \( \circ \) is max-min composition operator. The relation \( R \) is called first - order model of \( F(t). \) Further, if fuzzy relation \( R(t, t-1) \) of \( F(t) \) is independent of time \( t, \) that is to say, for different times \( t_1 \) and \( t_2, \)

\[ R(t_1, t_1 - 1) = R(t_2, t_2 - 1), \]

then \( F(t) \) is called a time invariant fuzzy time series, otherwise \( F(t) \) is time variant.

Definition 3: Suppose \( F(t-1) = A_i \) and \( F(t) = A_j. \) A fuzzy logical relationship can be defined as \( A_i \rightarrow A_j \) where \( A_i \) and \( A_j \) are called the left hand side and the right hand side of the fuzzy logical relationship respectively.

If there are fuzzy logical relationships is obtained from \( A_i \) as \( A_1 \rightarrow A_2, A_1 \rightarrow A_3, A_2 \rightarrow A_3, \) the fuzzy logical relationships are grouped into a fuzzy logical relationship group [19] as \( A_2 \rightarrow A_3, A_1, A_i. \)

3. Proposed method:

Step 1: Define the universe of discourse \( U, \) based on the range of available historical time series data by the following rules:

\[ U = [D_{min} - D_1, D_{max} + D_2], \]

where \( D_1 \) and \( D_2 \) are two proper positive numbers.

Step 2: Partition the universe of discourse in to equal length of intervals: \( u_1, u_2, \ldots, u_m. \) The number of intervals will be in accordance with the number of linguistic values (fuzzy sets) \( A_1, A_2, \ldots, A_m \) to be considered.

Step 3: Define fuzzy sets on the universe of discourse \( U. \) There is no restriction on determining how many linguistic variables can be fuzzy sets. Thus, the “enrollment” can be described by the fuzzy sets of \( A_1 = \) (not many), \( A_2 = \) (not too many), \( A_3 = \) (many), \( A_4 = \) (many), \( A_5 = \) (very many), \( A_6 = \) (too many), \( A_7 = \) (too many many). For simplicity, each fuzzy set \( A_i (i = 1, 2, \ldots, 7) \) is defined on seven intervals, which are \( u_1 = [d_1, d_2], u_2 = [d_2, d_3], u_3 = [d_3, d_4], u_4 = [d_4, d_5], \ldots, u_7 = [d_7, d_8]; \) thus, the fuzzy sets \( A_1, A_2, \ldots, A_7 \) are defined as follows:
\[ A_1 = \{1/u_1, 0.5/u_2, 0/u_3, 0/u_4, 0/u_5, 0/u_6, 0/u_7\} \]
\[ A_2 = \{0.5/u_1, 1/u_2, 0.5/u_3, 0/u_4, 0/u_5, 0/u_6, 0/u_7\} \]
\[ A_3 = \{0/u_1, 0.5/u_2, 1/u_3, 0.5/u_4, 0/u_5, 0/u_6, 0/u_7\} \]
\[ A_4 = \{0/u_1, 0/u_2, 0.5/u_3, 1/u_4, 0.5/u_5, 0/u_6, 0/u_7\} \]
\[ A_5 = \{0/u_1, 0/u_2, 0/u_3, 0.5/u_4, 1/u_5, 0.5/u_6, 0/u_7\} \]
\[ A_6 = \{0/u_1, 0/u_2, 0/u_3, 0/u_4, 0.5/u_5, 1/u_6, 0.5/u_7\} \]
\[ A_7 = \{0/u_1, 0/u_2, 0/u_3, 0/u_4, 0/u_5, 0.5/u_6, 1/u_7\} \]

**Step 4:** Fuzzify the historical data and establish the \( \pi \)-order fuzzy relationship \( A_{t-\pi}, A_{t-\pi+1}, \ldots, A_{t-2}, A_{t-1} \rightarrow A_t \), where \( \pi \geq 1 \) and \( t \geq 2 \) and where \( A_{t-1}, A_t \) denote the latest past in the current state and the next state respectively. This step aims to find an equivalent fuzzy set for each input data. If the collected time series data belongs to an interval \( u_{ij}, i = 1, 2, \ldots, 7 \), then it is fuzzified to the fuzzy sets \( A_i \).

**Step 5:** Determine \( \pi \)-order, \( \pi \geq 1 \) fuzzy logical relationship groups. All fuzzy relationships with the same current state are put together to form fuzzy relationship groups.

**Step 6:** We consider two cases to forecast the output values namely Markov state transition matrix method and aggregate method.

**Case (i) : (By Markov state transition matrix method)**

(a) Calculating forecasting values:

We define \( n \) states for each time step for \( n \) fuzzy sets to establish \( n \times n \) Markov state transition matrix. If state \( A_i \) makes a transition in to state \( A_j \) and passes another state \( A_k, i, j = 1, 2, \ldots, n \), then we can obtain the fuzzy logical relationship group. The transition probability of state \( A_1 \) is written as

\[ p_{ij} = \frac{P_{ij}}{T_{ij}}, \quad i, j = 1, 2, \ldots, n \]

where \( p_{ij} \) is the probability of transition from state \( A_i \) to \( A_j \) by one step. \( T_{ij} \) is the amount of data belonging to the \( A_i \) state. Then the transition probability matrix \( R \) of the state can be written as

\[
R = \begin{bmatrix}
p_{11} & \cdots & p_{1n} \\
\vdots & \ddots & \vdots \\
p_{n1} & \cdots & p_{nn}
\end{bmatrix}
\]

**Definition 3.2.1 [17]:** If \( p_{ij} \geq 0 \), then state \( A_j \) is reached from state \( A_i \). If the state \( A_1 \) is reached from state \( A_j \), then we say that the states \( i \) and \( j \) are communicate (i.e.) \( A_i \) communicates with \( A_j \). If a transition occurs from \( A_i \) to \( A_j \), with probability \( p_{ij} \geq 0, j = 1, 2, \ldots, n \), then \( \sum_j p_{ij} = 1 \).

If \( F(t-1) = A_n \), the process is defined to be in state \( A_i \) at time \( t-1 \); then forecasting of \( F(t) \) is conducted using the row vector \([P_{i1}, P_{i2}, \ldots, P_{in}]\). The forecasting of \( F(t) \) is equal to the weighted average of \( m_1, m_2, \ldots, m_n \), the midpoint of \( u_1, u_2, \ldots, u_n \). The expected forecasting values are obtained by the following Rules:

**Rule 1:** If the fuzzy logical relationship group of \( A_i \) is one-to-one (i.e., \( A_i \rightarrow A_k \), with \( P_{ij} = 1 \) and \( P_{ji} = 0, j \neq k \), then the forecasting of \( F(t) \) is \( m_k \), the midpoint of \( u_k \), according to the equation \( F(t) = m_kP_{ik} = m_k \).

**Rule 2:** If the fuzzy logical relationship group of \( A_i \) is one-to-many (i.e., \( A_i \rightarrow A_1, A_2, \ldots, A_n \), \( j = 1, 2, \ldots, n \)), when collected data \( Y(t-1) \) at time \( t-1 \) is in the state \( A_i \), then the forecasting of \( F(t) \) is equal to \( F(t) = m_1P_{i1} + m_2P_{i2} + \ldots + m_nP_{in} + Y(t-1) \), where \( m_1, m_2, \ldots, m_n, u_1, u_2, \ldots, u_n \) and \( Y(t-1) \) is substituted for \( m_j \) in order to take more information from the state \( A_j \) at time \( t-1 \).

(b) Adjusted forecasting values:

Adjust the tendency of the forecasting values. For any time series experiment, a large sample size is always necessary. Therefore, under a smaller sample size when modeling a fuzzy time series-Markov chain model, the derived Markov chain matrix is usually biased, and some adjustments for the forecasting values are suggested to revise the forecasting error. First, in a fuzzy logical relationship group where \( A_i \) communicates with \( A_k \) and is one-to-many, if a large state \( A_k \) is accessible from state \( A_i, i, j = 1, 2, \ldots, n \), then the forecasting value for \( A_k \) is usually underestimated because the lower state values are used for forecasting the value of \( A_k \). On the other hand, an overestimated value should be adjusted for forecasting value \( A_k \) because a smaller state \( A_k \) is accessible from \( A_i, i, j = 1, 2, \ldots, n \). Second, any transition that jumps more than two steps from one state to another state will derive a change-point forecasting value, so that it is necessary to make an adjustment to the forecasting value in order to obtain a smoother value. That is, if the data happens in the state \( A_i \) and then jumps forward to state \( A_{i+k} (k \geq 2) \) or jumps backward to state \( A_{i-k} (k \geq 2) \), then it is fuzzified to the fuzzy sets \( A_i \).
2), then it is necessary to adjust the trend of the pre-obtained forecasting value in order to reduce the estimated error. The adjusting rule for the forecasting value is described below.

**Rule 1:** If state \( A_i \) communicates with \( A_t \), starting in state \( A_t \) at time \( t-1 \) as \( F(t-1) = A_t \) and makes a transition into state \( A_{t+1} \) at time \( t \), then the adjusted forecasting value is \( F'(t) = F(t) + \left( \frac{l}{2} \right) \), where \( l \) is the length of the interval.

**Rule 2:** If state \( A_t \) communicates with \( A_i \), starting in state \( A_t \) at time \( t-1 \) as \( F(t-1) = A_i \) and makes a transition into state \( A_{t+1} \) at time \( t \), then the adjusted forecasting value is \( F'(t) = F(t) - \left( \frac{l}{2} \right) \).

**Rule 3:** If the current state is in state \( A_i \) and makes a forward transition into state \( A_j \) at time \( t \) and \( A_i \) communicates with \( A_j \), then the adjusted forecasting value is \( F'(t) = F(t) + 2\left( \frac{l}{2} \right) \).

**Rule 4:** If the current state is in state \( A_t \) at time \( t-1 \) as \( F(t-1) = A_t \) and makes a backward transition into state \( A_i \) at time \( t \) and \( A_t \) communicates with \( A_i \), then the adjusted forecasting value is \( F'(t) = F(t) - 2\left( \frac{l}{2} \right) \).

**Rule 5:** If the current state is in state \( A_t \) at time \( t-1 \) as \( F(t-1) = A_t \) and makes a jump forward transition into state \( A_{t+i} \) at time \( t \) \((1 < s \leq n-i) \) then the adjusted forecasting value is \( F'(t) = F(t) + \left( \frac{s}{2} \right) \) s.

**Rule 6:** If the current state is in state \( A_t \) at time \( t-1 \) as \( F(t-1) = A_t \) and makes a jump backward transition into state \( A_{i-s} \) at time \( t \) \((1 < s \leq i) \) then the adjusted forecasting value is \( F'(t) = F(t) - \left( \frac{s}{2} \right) \) s.

**Case (ii): (By Aggregate method) Calculating forecasting values:**

To improve the forecasting accuracy, we discuss a new forecasting method which aggregates the overall information and latest variation scheme to calculate the forecasted value. As shown in equation (1), \( w_1 \) and \( w_2 \) are adaptive weights for overall information of the fuzzy relationships and latest information respectively. For each group of the fuzzy rule, the forecasted value consists of two weighted parts on overall information and latest information respectively. Here by, the forecasted value of enrollments is represented as:

\[
\text{Forecasted value} = w_1 \times \text{overall information} + w_2 \times \text{latest information}, \quad (1)
\]

where \( w_1 + w_2 = 1 \) and \( 0 \leq w_1, w_2 \leq 1 \). Without loss of generality, we assume \( w_1 \) and \( w_2 \) are equally weighted. Initially, we consider training data which has known current state and next state. In equation (1), the overall information can be decided by the fuzzy groups created in step 5.

Based on the adopted method of defuzzification by Chen (1996), the defuzzified value can be computed by the midpoint of the next state for each fuzzy relationship. By applying seven intervals in step 2, the midpoint of each interval \( u_i \) can be calculated as follows: \( m_i = (b_{s_i} - b_{e_i})/2 \), where \( 1 \leq t \leq 7 \) and \( u_i \) is bounded within \( (b_{s_i}, b_{e_i}) \). If a fuzzy relationship group consists of more than one fuzzy relationship, the value of overall information is the average of the corresponding midpoints of all intervals with respect to all linguistic values existing in the next states of all fuzzy relationships. Assume a first-order fuzzy relationship group is \( A_{t-1} \rightarrow A_{t+1}, A_{t+2}, ..., A_{t+k} \), and \( m_{t-1}, m_{t+2}, ..., m_{t+k} \) are the midpoints of linguistic values \( A_{t+1}, A_{t+2}, ..., A_{t+k} \), respectively. Then, the value of overall information is calculated as follows:

\[
\text{overall information} = \frac{m_{t+1} + m_{t+2} + \ldots + m_{t+k}}{k} \quad (2)
\]

The latest information is derived by the latest fuzzy variation scheme. This scheme is an estimating scheme determined by the next state and the latest past in the current state. Assume a \( \pi \)-order fuzzy relationship is \( A_{t-\pi}, A_{t-\pi+1}, ..., A_{t-2}, A_{t-1} \rightarrow A_t \), where \( \pi \geq 1 \) and \( t \geq 2 \), where \( A_{t-1} \) and \( A_t \) denote the latest past in the current state and the next state, respectively. Here, \( m_{t-1} \) and \( m_t \) are midpoints of the fuzzy intervals \( u_{t-1} \) and \( u_t \) with respective to \( A_{t-1} \) and \( A_t \) where \( u_{t-1} = (b_{s_{t-1}}, b_{e_{t-1}}) \), \( u_t = (b_{s_t}, b_{e_t}) \). As shown in equation (3), the latest variation scheme calculates fuzzy difference between \( A_{t-1} \) and \( A_t \) using \( m_{t-1} \) and \( m_t \); then the fuzzy difference should be normalized by dividing \( m_{t-1} + m_t \). The complete latest variation scheme is formulated as follows:

\[
\text{Latest information} = \frac{b_{e_{t-1}} - b_{s_{t-1}}}{2} \times \frac{m_{t-1} - m_{t}}{m_{t} + m_{t-1}} \quad (3)
\]

Here we take the model of the first-order as framing rules on fuzzy relationships.

**Step 7:** Choose \( F_i \) by the rule (i) If \( |A_{i-1} - F_i| < |A_{i-1} - F_i| \) choose \( F_i = F_i \) (or) rule (ii) If \( |A_{i-1} - F_i| > |A_{i-1} - F_i| \) choose \( F_i = F_i \)

**Step 8:** Choose \( \alpha \in (0, 1) \). Make an error analysis as follows:
Final forecasted value = \frac{Actual\ value + New\ forecasted\ value}{2} - \propto

The AFER is used to measure the accuracy as a percentage as follows:

\[ \text{AFER} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{A_i - F_i}{A_i} \right| \times 100. \]

4. Results and Analysis

Step 1: Define universe of discourse \( U \) and partition it into seven equal-length intervals, The collected data is shown in the second column of table 1; we have the enrollments of the university from 1971 to 1992 with \( D_{\text{min}} = 13055 \) and \( D_{\text{max}} = 19337 \). We choose \( D_1 = 55 \) and \( D_2 = 663 \). Thus, \( U = [13000, 20000] \).

Step 2: \( U \) is partitioned into seven intervals with \( u_1 = [13000, 14000] \), \( u_2 = [14000, 15000] \), \( u_3 = [15000, 16000] \), \( u_4 = [16000, 17000] \), \( u_5 = [17000, 18000] \), \( u_6 = [18000, 19000] \), \( u_7 = [19000, 20000] \). The mid-values of the intervals are \( m_1 = 13500 \), \( m_2 = 14500 \), \( m_3 = 15500 \), \( m_4 = 16500 \), \( m_5 = 17500 \), \( m_6 = 18500 \), \( m_7 = 19500 \).

Step 3: The fuzzy sets corresponding to the enrollments of the university from 1971 to 1992 are given in table 1.

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual Enrollment</th>
<th>Fuzzy sets</th>
<th>Year</th>
<th>Actual Enrollment</th>
<th>Fuzzy sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971</td>
<td>13055</td>
<td>( A_1 )</td>
<td>1982</td>
<td>15433</td>
<td>( A_3 )</td>
</tr>
<tr>
<td>1972</td>
<td>13563</td>
<td>( A_1 )</td>
<td>1983</td>
<td>15497</td>
<td>( A_3 )</td>
</tr>
<tr>
<td>1973</td>
<td>13867</td>
<td>( A_1 )</td>
<td>1984</td>
<td>15145</td>
<td>( A_3 )</td>
</tr>
<tr>
<td>1974</td>
<td>14696</td>
<td>( A_2 )</td>
<td>1985</td>
<td>15163</td>
<td>( A_3 )</td>
</tr>
<tr>
<td>1975</td>
<td>15460</td>
<td>( A_3 )</td>
<td>1986</td>
<td>15984</td>
<td>( A_3 )</td>
</tr>
<tr>
<td>1976</td>
<td>15311</td>
<td>( A_3 )</td>
<td>1987</td>
<td>16859</td>
<td>( A_4 )</td>
</tr>
<tr>
<td>1977</td>
<td>15603</td>
<td>( A_3 )</td>
<td>1988</td>
<td>18150</td>
<td>( A_6 )</td>
</tr>
<tr>
<td>1978</td>
<td>15861</td>
<td>( A_3 )</td>
<td>1989</td>
<td>18970</td>
<td>( A_6 )</td>
</tr>
<tr>
<td>1979</td>
<td>16807</td>
<td>( A_4 )</td>
<td>1990</td>
<td>19328</td>
<td>( A_7 )</td>
</tr>
<tr>
<td>1980</td>
<td>16919</td>
<td>( A_4 )</td>
<td>1991</td>
<td>19337</td>
<td>( A_7 )</td>
</tr>
<tr>
<td>1981</td>
<td>16388</td>
<td>( A_4 )</td>
<td>1992</td>
<td>18876</td>
<td>( A_6 )</td>
</tr>
</tbody>
</table>

Step 4:

<table>
<thead>
<tr>
<th>Year</th>
<th>Fuzzy sets</th>
<th>First-order</th>
<th>Three-order</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971</td>
<td>( A_1 )</td>
<td>( A_1 \rightarrow A_1 )</td>
<td></td>
</tr>
<tr>
<td>1972</td>
<td>( A_1 )</td>
<td>( A_1 \rightarrow A_1 )</td>
<td>( A_1, A_1 \rightarrow A_2 )</td>
</tr>
<tr>
<td>1973</td>
<td>( A_1 )</td>
<td>( A_1 \rightarrow A_1 )</td>
<td></td>
</tr>
<tr>
<td>1974</td>
<td>( A_2 )</td>
<td>( A_1 \rightarrow A_2 )</td>
<td>( A_1, A_1 \rightarrow A_2 )</td>
</tr>
<tr>
<td>1975</td>
<td>( A_3 )</td>
<td>( A_2 \rightarrow A_3 )</td>
<td>( A_1, A_1 \rightarrow A_3 )</td>
</tr>
<tr>
<td>1976</td>
<td>( A_3 )</td>
<td>( A_3 \rightarrow A_3 )</td>
<td>( A_1, A_2 \rightarrow A_3 )</td>
</tr>
<tr>
<td>1977</td>
<td>( A_3 )</td>
<td>( A_3 \rightarrow A_3 )</td>
<td>( A_2, A_3 \rightarrow A_3 )</td>
</tr>
<tr>
<td>1978</td>
<td>( A_3 )</td>
<td>( A_3 \rightarrow A_3 )</td>
<td>( A_3, A_3 \rightarrow A_3 )</td>
</tr>
</tbody>
</table>
1979 | $A_4$ | $A_3 \rightarrow A_4$ | $A_3, A_4 \rightarrow A_4$
1980 | $A_4$ | $A_4 \rightarrow A_4$ | $A_3, A_4 \rightarrow A_4$
1981 | $A_4$ | $A_4 \rightarrow A_4$ | $A_3, A_4, A_4 \rightarrow A_4$
1982 | $A_3$ | $A_4 \rightarrow A_3$ | $A_4, A_4, A_4 \rightarrow A_3$
1983 | $A_3$ | $A_3 \rightarrow A_3$ | $A_4, A_4$ $A_3 \rightarrow A_3$
1984 | $A_3$ | $A_3 \rightarrow A_3$ | $A_4, A_3$ $A_3 \rightarrow A_3$
1985 | $A_3$ | $A_3 \rightarrow A_3$ | $A_3, A_3$ $A_3 \rightarrow A_3$
1986 | $A_3$ | $A_3 \rightarrow A_3$ | $A_3, A_3$ $A_3 \rightarrow A_3$
1987 | $A_4$ | $A_3 \rightarrow A_4$ | $A_3, A_3$ $A_3 \rightarrow A_4$
1988 | $A_6$ | $A_4 \rightarrow A_6$ | $A_3, A_4$ $A_6 \rightarrow A_6$
1989 | $A_6$ | $A_6 \rightarrow A_6$ | $A_3, A_4$ $A_6 \rightarrow A_6$
1990 | $A_7$ | $A_6 \rightarrow A_7$ | $A_4, A_6$ $A_6 \rightarrow A_7$
1991 | $A_7$ | $A_7 \rightarrow A_7$ | $A_6, A_7\rightarrow A_7$
1992 | $A_6$ | $A_7 \rightarrow A_6$ | $A_6, A_7\rightarrow A_6$

Step 5:

**Table 3:** The first-order fuzzy relationship groups on enrollments

<table>
<thead>
<tr>
<th>Groups</th>
<th>First-order fuzzy relationship groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>$A_1 \rightarrow A_1, A_2$</td>
</tr>
<tr>
<td>$G_2$</td>
<td>$A_1 \rightarrow A_1$</td>
</tr>
<tr>
<td>$G_3$</td>
<td>$A_1 \rightarrow A_3, A_4$</td>
</tr>
<tr>
<td>$G_4$</td>
<td>$A_4 \rightarrow A_5, A_5, A_6$</td>
</tr>
<tr>
<td>$G_5$</td>
<td>$A_6 \rightarrow A_6, A_7$</td>
</tr>
<tr>
<td>$G_6$</td>
<td>$A_7 \rightarrow A_7, A_8$</td>
</tr>
</tbody>
</table>

**Table 4:** The three-order fuzzy relationship groups on enrollments

<table>
<thead>
<tr>
<th>Groups</th>
<th>Three-order fuzzy relationship groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>$A_1, A_2, A_2 \rightarrow A_2$</td>
</tr>
<tr>
<td>$G_2$</td>
<td>$A_1, A_2, A_2 \rightarrow A_3$</td>
</tr>
<tr>
<td>$G_3$</td>
<td>$A_1, A_3, A_3 \rightarrow A_3$</td>
</tr>
<tr>
<td>$G_4$</td>
<td>$A_3, A_3, A_3 \rightarrow A_3$</td>
</tr>
<tr>
<td>$G_5$</td>
<td>$A_3, A_3, A_3 \rightarrow A_3$</td>
</tr>
<tr>
<td>$G_6$</td>
<td>$A_3, A_3, A_3 \rightarrow A_3$</td>
</tr>
<tr>
<td>$G_7$</td>
<td>$A_4, A_4, A_4 \rightarrow A_4$</td>
</tr>
<tr>
<td>$G_8$</td>
<td>$A_4, A_4, A_4 \rightarrow A_4$</td>
</tr>
<tr>
<td>$G_9$</td>
<td>$A_4, A_4, A_4 \rightarrow A_4$</td>
</tr>
<tr>
<td>$G_{10}$</td>
<td>$A_4, A_4, A_4 \rightarrow A_4$</td>
</tr>
<tr>
<td>$G_{11}$</td>
<td>$A_4, A_4, A_4 \rightarrow A_4$</td>
</tr>
<tr>
<td>$G_{12}$</td>
<td>$A_4, A_4, A_4 \rightarrow A_4$</td>
</tr>
<tr>
<td>$G_{13}$</td>
<td>$A_4, A_4, A_4 \rightarrow A_4$</td>
</tr>
</tbody>
</table>
Step 6:

Case(i):

Using the fuzzy logical relationship group in Table 2, the transition probability matrix R may be obtained.

\[
R = \begin{bmatrix}
\frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{7}{9} & \frac{2}{9} & 0 & 0 & 0 \\
0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\]

Adjust the tendency of the forecasting values. The relationships between the states are analyzed in Figure 1. It is clear that states 3 and 4 communicate with each other, thus an adjust value should be considered, or vice versa. By contrast, state 6 and state 7 also communicate with each other, but in the end these states uncertainty in relation to the future trend is larger and unknown; thus, we do not adjust the value.

Figure 1: Transition process for enrollment forecasting

For case (i):

Suppose we want to adjust the forecasted value for the year 1979. According to the Rule 2 of case (i) (a), the forecasting value of the year 1979.

\[
F(1979) = \left(\frac{7}{9}\right)Y(1978) + \left(\frac{2}{9}\right)m_A
\]

\[
= \left(\frac{7}{9}\right)(15861) + \left(\frac{2}{9}\right)16500
\]

\[
= 16003.
\]

The corresponding fuzzy logical relationship is \(A_3 \rightarrow A_4\). According to figure 1, the states 3 and 4 communicate with each other. Then by Rule 3 of case (i) (b), the adjusted forecasting value is

\[
F'(1979) = F(1979) + 2 \left(\frac{l}{2}\right), \text{ where } l = 1000
\]

\[
= 16003 + 1000
\]

\[
= 17003.
\]

For case (ii):

Suppose we want to forecast the enrollment of year 1975. Based on table 1, we see that the linguistic enrollment of current state at 1974 is \(A_2\). In table 3, we find that a fuzzy relationship \(A_2 \rightarrow A_3\) in group \(G_2\) appears the same linguistic value of the current state \(A_2\). The maximum membership values of the fuzzy sets \(A_2\) and \(A_3\) occur at intervals \(u_2\) and \(u_3\), respectively, where \(u_2 = (b_{s2}, b_{e2})\) and \(u_3 = (b_{s3}, b_{e3})\). From step 2, we obtained \(b_{s3} = 14000, b_{e2} = 15000, b_{s3} = 15000\) and \(b_{e3} = 16000\). The midpoints of the intervals \(u_2\) and \(u_3\) are \(m_2 = 14500\) and \(m_3 = 15500\), respectively, where \(m_2 = \frac{1}{2}(14000 + 15000)\) and \(m_3 = \frac{1}{2}(15000 + 16000)\). The overall information of year 1975 is equal to \(m_3\), that is overall information \(=15500\). According to equation (3), by setting \(b_{s3} = b_{s3}, b_{e3} = b_{e3}, m_{i-1} = m_2, m_i = m_3, u_{i-1} = u_2\) and \(u_i = u_3\), we can calculate the value of the latest information on the enrollment of year 1975 as follows:

\[
Latest\ information = b_{s3} + \frac{b_{s3} - b_{e3}}{2} \left(\frac{m_3 - m_2}{m_3 + m_2}\right)
\]

\[
= 15000 + \frac{15000 - 14000}{15000 + 14000} \times \frac{15500 - 15000}{15500 + 15000}
\]

\[
= 15500.
\]
Moreover, assume we want to forecast the enrollment of year 1982, we can see that the current state of the enrollment of year 1981 is \( A_4 \) in table 2, we can find that three-order fuzzy relationships \( A_4 \rightarrow A_4, A_4 \rightarrow A_3, \) and \( A_4 \rightarrow A_6 \) in group \( G_4 \) appear the same current state \( A_4 \). Based on equations (1) to (3), the forecasted enrollment of the year 1982 can be calculated as follows:

\[
\text{Forecasted value} = w_1 \times \frac{m_4 + m_3 + m_6}{3} + w_2 \times \left( b_5 \times \frac{b_3 - b_5}{2} \times \frac{m_3 - m_4}{m_3 + m_4} \right)
\]

\[
= 0.5 \times \frac{16500 + 15500 + 18500}{3} + 0.5 \times \frac{16000 - 15000}{2} \times \frac{15500 - 15000}{15500 + 15000}
\]

\[
= 15908.
\]

The forecasted enrollments of the first-order fuzzy relationships are listed in table 5. In the case (i), adjust the forecasted value using the rules concerned in step 6 of case (i) (b). Tabulate the forecasting values which are nearer to the actual values from the two cases.

**Table 5: Enrollment forecasting**

<table>
<thead>
<tr>
<th>Year</th>
<th>Historical data</th>
<th>By case(i)</th>
<th>By case(ii)</th>
<th>New forecasted value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( A_i )</td>
<td>Forecasted value ( F_i )</td>
<td>Adjusted forecasted value ( F_i' )</td>
<td>Forecasted value ( F_i' )</td>
</tr>
<tr>
<td>1971</td>
<td>13055</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1972</td>
<td>13563</td>
<td>13537</td>
<td>13537</td>
<td>13500</td>
</tr>
<tr>
<td>1973</td>
<td>13867</td>
<td>13875</td>
<td>13875</td>
<td>13500</td>
</tr>
<tr>
<td>1974</td>
<td>14696</td>
<td>14078</td>
<td>14457</td>
<td>14009</td>
</tr>
<tr>
<td>1975</td>
<td>15460</td>
<td>15500</td>
<td>15500</td>
<td>15258</td>
</tr>
<tr>
<td>1976</td>
<td>15311</td>
<td>15691</td>
<td>15691</td>
<td>15500</td>
</tr>
<tr>
<td>1977</td>
<td>15603</td>
<td>15575</td>
<td>15575</td>
<td>15500</td>
</tr>
<tr>
<td>1978</td>
<td>15861</td>
<td>15802</td>
<td>15802</td>
<td>15500</td>
</tr>
<tr>
<td>1979</td>
<td>16807</td>
<td>16003</td>
<td>17003</td>
<td>16008</td>
</tr>
<tr>
<td>1980</td>
<td>16919</td>
<td>16904</td>
<td>16904</td>
<td>16416</td>
</tr>
<tr>
<td>1981</td>
<td>16388</td>
<td>16960</td>
<td>16960</td>
<td>16416</td>
</tr>
<tr>
<td>1982</td>
<td>15433</td>
<td>16694</td>
<td>15694</td>
<td>15908</td>
</tr>
<tr>
<td>1983</td>
<td>15497</td>
<td>15670</td>
<td>15670</td>
<td>15500</td>
</tr>
<tr>
<td>1984</td>
<td>15145</td>
<td>15720</td>
<td>15720</td>
<td>15500</td>
</tr>
<tr>
<td>1985</td>
<td>15163</td>
<td>15446</td>
<td>15446</td>
<td>15500</td>
</tr>
<tr>
<td>1986</td>
<td>15984</td>
<td>15460</td>
<td>15460</td>
<td>15500</td>
</tr>
<tr>
<td>1987</td>
<td>16859</td>
<td>16909</td>
<td>17099</td>
<td>16008</td>
</tr>
<tr>
<td>1988</td>
<td>18150</td>
<td>16930</td>
<td>17930</td>
<td>17431</td>
</tr>
<tr>
<td>1989</td>
<td>18970</td>
<td>18600</td>
<td>18600</td>
<td>18500</td>
</tr>
<tr>
<td>1990</td>
<td>19328</td>
<td>19147</td>
<td>19647</td>
<td>19006</td>
</tr>
<tr>
<td>1991</td>
<td>19337</td>
<td>18914</td>
<td>18914</td>
<td>19000</td>
</tr>
<tr>
<td>1992</td>
<td>18876</td>
<td>18919</td>
<td>18919</td>
<td>18493</td>
</tr>
</tbody>
</table>

**Step 7:** Using AFER, we calculated the value of the proposed method. This calculated value and some other values available in the literature related to this work are tabulated in table 6.
Table 6: Comparison of forecasting errors for six types of methods

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AFER (%)</td>
<td>3.22</td>
<td>1.86</td>
<td>1.7236</td>
<td>1.5587</td>
<td>1.53</td>
<td>0.5065</td>
</tr>
</tbody>
</table>

4. Conclusion

In this paper, we have presented a model in academic enrollments based on Markov chain with aggregated fuzzy relationships. First, we find out the relationship matrix using Markov analysis and adjust the forecasted values. Secondly, we studied the effect of forecasting accuracy relates to the overall information and the latest variation. From the empirical study of forecasting enrollments of students of the University of Alabama, the experimental results show that the proposed model gets higher forecasting accuracy than any existing models tabulated in table 6. Further study can be made on the nature of states of the Markov matrix. Based on the behaviour of the states the fuzzy relationships may be aggregated.

References