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EXTON'S HYPERGEOMETRIC FUNCTIONS OF THREE VARIABLES IN TERMS OF INTEGRAL LAPLACE TRANSFORM

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Abstract:

It has been obtained some results on Exton's Hypergeometric functions associated with integral representation, Next our aim in this Paper is to evaluate Exton's Hypergeometric functions in terms of integral Laplace transform. Also some of Known results are obtained as special cases of our definitions.

Keywords: Exton's Hypergeometric functions, Beta and Gamma functions, Hypergeometric functions of three variables.

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1. Introduction

Accoundingly in the theory of in the theory of $hypergeometric$ function of several variables, a remarkable large number of triple hypergeometric functions have been introduced and investigated. A comprehensive table of 205 distinct triple hypergeometric function is provided in the work of Srivastava and Karlsson [5, chapter 3].Out of these 205 distinct triple hypergeometric function, Lauricella [6, p.14] introduced Fourteen complete triple hypergeometric function of the second order, Next to the three variables case of the n – variables Lauricella function see ([6, p.113]. Thus Exton [3] introduced 20 distinct triple hypergeometric function which he denoted by $X_1,\ldots,X_{20}$ and investigated their twenty Laplace integral representation whose kernals include the confluent hypergeometric function ${}_0F_1, {}_1F_1$, and the Humbert hypergeometric function ϕ_2, Ψ_2,of two variables, but we take here only the definition of X_2 , X_4 , X_7 , X_8 and X_{12}, are as follows (cf.[2,3]).

(1.1) $$X_2(\alpha,\beta;\gamma,\delta,\lambda;x,y,z) = \sum_{m,n,p=0}^{\infty} \frac{(\alpha)_{2m+2n+p}\,(\beta)_p}{(\gamma)_m(\delta)_n(\lambda)_p}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}.$$

(1.2) $$X_4(\alpha,\beta;\gamma,\delta,\lambda;x,y,z) = \sum_{m,n,p=0}^{\infty} \frac{(\alpha)_{2m+n+p}\,(\beta)_{n+p}}{(\gamma)_m(\delta)_n(\lambda)_p}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}.$$

(1.3) $$X_7(\alpha,\beta_1,\beta_2;\gamma,\delta;x,y,z) = \sum_{m,n,p=0}^{\infty} \frac{(\alpha)_{2m+n+p}\,(\beta_1)_n\,(\beta_2)_p}{(\gamma)_m(\delta)_{n+p}}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}.$$

(1.4) $$X_8(\alpha,\beta_1,\beta_2;\gamma,\delta,\lambda;x,y,z) = \sum_{m,n,p=0}^{\infty} \frac{(\alpha)_{2m+n+p}\,(\beta_1)_n\,(\beta_2)_p}{(\gamma)_m(\delta)_n(\lambda)_p}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}.$$

(1.5) $$X_{12}(\alpha,\beta;\gamma,\delta,\lambda;x,y,z) = \sum_{m,n,p=0}^{\infty} \frac{(\alpha)_{2m+n}(\beta)_{n+2p}}{(\gamma)_m(\delta)_n(\lambda)_p}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}.$$

2. Main results

It has seen some results on Exton's Hypergeometric functions related with integral Laplace transform see ref. [2, 3]

$$(2.1)\qquad X_2(\alpha,\beta;\gamma,\delta,\lambda;x,y,z)=\frac{1}{\Gamma(\alpha)}\int_0^1 e^{-t}\,\mathrm{t}^{\alpha-1}\,{}_0F_1\ (-;\gamma;xt^2)$$

$${}_0F_1\ (-;\delta;yt^2)\mathrm{t}^{\alpha-1}\,{}_0F_1\ (\beta;\lambda;zt)dt\,.$$

$$(2.2)\qquad X_4(\alpha,\beta;\gamma,\delta,\lambda;x,y,z)=\frac{1}{\Gamma(\alpha)}\int_0^1 e^{-t}\,\mathrm{t}^{\alpha-1}\,{}_0F_1\ (-;\gamma;xt^2)$$

$$\Psi_2(\beta;\delta,\lambda;ys,zs)dt$$

$$(2.3)\qquad X_7(\alpha,\beta_1,\beta_2;\gamma,\delta;x,y,z)=\frac{1}{\Gamma(\alpha)}\int_0^1 e^{-t}\,\mathrm{t}^{\alpha-1}\,{}_0F_1\ (-;\gamma;xt^2)$$

$$\phi_2(\beta_1,\beta_2;\delta;yt,zt)dt$$

$$(2.4)\quad X_8(\alpha,\beta_1,\beta_2;\gamma,\delta,\lambda;x,y,z)=\frac{1}{\Gamma(\alpha)}\int_0^1 e^{-t}\,t^{\alpha-1}\,{}_0F_1\ (-;\gamma;xt^2)$$

$${}_1F_1\ (\beta_1,;\delta;yt)\,{}_1F_1\ (\beta_2;\lambda;zt)dt$$

$$(2.5)\quad X_{12}(\alpha,\beta;\gamma,\delta,\lambda;x,y,z)=\frac{1}{\Gamma(\alpha)\Gamma(\beta)}\int_0^\infty\int_0^1 e^{-t}\,t^{\alpha-1}\mathrm{s}^{\beta-1}\,{}_0F_1\ (-;\gamma;xt^2)$$

$${}_0F_1\ (-;\delta;yst)\mathrm{t}^{\alpha-1}\,{}_0F_1\ (\beta;\lambda;zs^2)dtds\,.$$

3. Proof of main results

Proof of (2.1) we first consider (1.1) then apply gamma function and change the order of integration and summation, therefore these function see (ref. [1], p.19-22) are defined by

$$(3.1)\qquad (\lambda)_n=\frac{\Gamma(\lambda)}{\Gamma(\lambda+n)},\qquad \mathrm{Re}\ (z)>0$$

$$(3.2)\qquad \Gamma(z)\ =\int_0^1 t^{z-1}\ \mathrm{e}^{\mathrm{t}}dt,\qquad \mathrm{Re}\ (\lambda)>0$$

On the R.H.S. of said result (1.1) which yields

$$(3.3)\qquad X_2(\alpha,\beta;\gamma,\delta,\lambda;x,y,z)$$

$$=\frac{1}{\Gamma(\alpha)}\int_0^1 e^{-t}\,t^{\alpha-1}\left\{\sum_{m,n,p=0}^{\infty}\frac{(xt^2)^m}{(\gamma)_m m!}\frac{(yt^2)^n}{(\delta)_m n!}\frac{(zt)^p}{(\lambda)_p p!}\right\}dt$$

Using Gauss Hypergeometric function see (ref. [1], pp. 29)

$$(3.4) \qquad {}_2F_1\ [a,b;\ c;\ z] = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}, \qquad C \neq 0,-1,-2,-3\ \ldots,$$

In above results (3.3). Hence which gives the complete proof of (2.1)

Similarly proof of each of each results (2.2) to (2.5) is much akin to that of first main results (2.1) which we have already presented in a reasonably detailed manner, but instead of (3.4) we use another corresponding two results mentioned in confluent hypergeometric function of two variables out of twenty results which are given by Erde$'$lyi et al ((1953), Vol. 1, pp. 225-228 and also ref. [1], p. 58-59) for the proof of (2.2) to (2.3) are defined by

$$(3.5) \qquad \Psi_2(\alpha;\gamma,\gamma';x,y) = \sum_{m,n=0}^{\infty} \frac{(\alpha)_{m+n}}{(\gamma)_m(\gamma')_n} \frac{x^m}{m!}\frac{y^n}{n!}, \qquad |x|<\infty, |y|<\infty$$

$$(3.6) \qquad \phi_2(\beta,\beta';\gamma;x,y) = \sum_{m,n,0}^{\infty} \frac{(\beta)_m(\beta')_n}{(\gamma)_{m+n}} \frac{x^m}{m!}\frac{y^n}{n!}, \qquad |x|<\infty, |y|<\infty$$

Hence which gives complete proof of such results

4. Special cases

An important special cases which we derived from the definition (1.1) to (1.5) are

(4.1) $\quad Setting\ \alpha = \gamma = \delta = \lambda = 0\ and\ x, y = 0$

$in\ the\ definition$ (1.1) to (1.5) $can\ be$ rewritten as

$$(4.2) \qquad X_k(-,-;\gamma,-,-;x) = \sum_{n=0}^{\infty} \frac{x^m}{(\gamma)_m\, m!}\ {}_0F_1\ (-;\gamma;x)$$

$$where\ k = 2,4,7,8,12\ ;\ \gamma \neq 0,-1,-2,$$

In view see (ref. [1], p.37, eq. (9)) thus R.H.S of above result can also be rewritten as

$$(4.3) \qquad = \operatorname*{limt}_{|\alpha|\to\infty}\ {}_1F_1\ \left(\alpha\ ;\ \gamma\ ;\ \frac{z}{\alpha}\right)$$

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