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SPECIFIED FLOW IN A NON LINEAR CAPACITATED TRANSPORTATION PROBLEM

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ABSTRACT

In the present paper, anon linear transportation problem with specified flow is studied. I fin addition to the flow constraint, the minimum requirement of each destination is also specified then the situation arises of distributing at minimum cost a certain commodity produced in a country, after keeping reserve stocks, to various states with minimum requirement of each state specified. Are lated transportation problem is formed in which the flow constraint is replaced by two extra destinations, one for supplementing the total flow up to the specified level, and the other for identifying the supply points preferred to keeper serves. Optimal basic feasible solution of the related transportation problem so formulate dis shown to give an optimal solution of the given problem. A numerical example is included in support of theory.

Keywords: optimum time cost trade off, capacityated transportation problem, non linear transportation problem, specified flow.

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INTRODUCTION:

Animportantclassoftransportationproblem consistsofcapacitated transportation problem. Ifthetotalflowinatransportationproblem withboundsonrim conditionsisalsospecified, the resulting problem makes the transportation problem more realistic. Moreover, if the total capacity of each route is also specified then optimal solution of such problems is of greater importancewhichgivesrisetoacapacitated transportation problem. ManyresearcherslikeA.K Bit et.al. [6], K.Dahiya et.al. [7] Have contributed in this field.

In1976, Bhatiaet.al. [5] Providedthetimecosttradeoffpairsinalineartransportation problem.Thenin1994, Basuet.al. [4]Developedanalgorithm fortheoptimumtimecosttrade offpairsinafixedchargelineartransportationproblem givingsameprioritytocostaswellas time.Aroraet.al. [3]Studiedtimecosttradeoffpairsin anindefinitequadratictransportation problemwith restricted flow.

Inthispaper, acapacitatednon lineartransportationproblem with bounds on rimconditionsgivingsameprioritytocostandtimeis studiedalongwithspecificationonthe totalflow.Analgorithm toidentifytheefficientcosttimetradeoffpairsfortheproblem is developed.

PROBLEM FORMULATION:

Consider a non linear capacitated transportation problem given by

(P1):

$$\min z = \left[\frac{\sum_{i\in I} \sum_{j\in J} c_{ij} x_{ij}}{\sum_{i\in I} \sum_{j\in J} d_{ij} x_{ij}} + \frac{\sum_{i\in I} F_i}{\sum_{i\in I} G_i} \right]$$

Subject to $a_i \le \sum_{i\in I} X_{ij} \le A_i \ \forall\ i \in I$ 1.1

$b_j \le \sum_{j\in J} X_{ij} \le B_j \ \forall j \in J$ 1.2

And integers $\forall i \in I, j \in J$ 1.3

I = {1, 2, m} is the index set of m origins.

J = {1, 2, n} is the index set of n destinations

I= number of units transported from i^t origin to j^{th} destination.

C_{ij} = cost of transporting one unit of commodity from i^{th} origin to j^{th} destination.

$l_{ij}$and u_{ij} are the bounds on number of units to be transported from i^{th} origin to j^{th} destination.

a_i is the availability at the i^{th} origin, $i \in I$

b_j is the bounds on the demand at the jth destination, $j \in J$

t_{ij} is the time of transporting goods from i^{th} origin to j^{th}destination.

F_i is the fixed cost associated with i^{th} origin.

$$\sum_{i \in I} \sum_{j \in J} x_{ij} = P \quad , \; x_{ij} \geq 0; \forall (\mathrm{i}, \mathrm{j}) \in \mathrm{I} \times \mathrm{J}$$

Where $\sum_{j \in J} r_j < P < \sum_{i \in I} a_i$, r_j being the minimum requirement specified for the j^{th} destination.

.The resulting on linear capacitated transportation problem with specified flow is

$$(\mathrm{P}_1)\text{: } \min z = \left[\frac{\sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij}}{\sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij}} + \frac{\sum_{i \in I} F_i}{\sum_{i \in I} G_i} \right]$$

Subject to

$$\left.\begin{aligned}
&\sum_{i \in I} X_{ij} \geq a_i \;\; \forall \; i \in I \\
&\sum_{j \in J} X_{ij} \geq b_j \;\; \forall j \in J \\
&l_{ij} \leq x_{ij} \leq u_{ij} \qquad \text{and integers} \;\; \forall i \in I, j \in J \\
&\sum_{i \in I} \sum_{j \in J} X_{ij} \; = \sum_{j \in J} r_j < P < \sum_{i \in I} a_i \;\; x_{ij} \geq 0 \;\; i \in I, j \in J \\
&(\mathrm{P}_2)\text{: } \min z = \left[\frac{\sum_{i \in I} \sum_{j \in J} c'_{ij} x_{ij}}{\sum_{i \in I} \sum_{j \in J} d'_{ij} x_{ij}} + \frac{\sum_{i \in I} F'_i}{\sum_{i \in I} G'_i} \right]
\end{aligned}\right\} \quad \ldots\ldots\ldots.(1)$$

Subject to (1.1), (1.2), (1.3) and

To solve the problem (P_2), we first convert it into related problem (P'_2) given below.

$$(\mathrm{P}'_2)\text{: } \min z = \left[\frac{\sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij}}{\sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij}} + \frac{\sum_{i \in I} F_i}{\sum_{i \in I} G_i} \right]$$

Subject to

$$\sum_{j\in J'} y_{ij} = a_i{}' \forall i \in I'$$

$$\sum_{i\in I'} y_{ij} = b_j{}' \forall j \in J'$$

$$l_{ij} \le y_{ij} \le u_{ij} \forall i \in I, j \in J$$

$$0 \le y_{m+l,j} \le \sum_{i\in I} u_{ij} - b_j ; \forall j \in J$$

$$0 \le y_{i,n+1} \le \sum_{j\in J} u_{ij} - a_i \forall i \in I$$

$y_{m+1,n+1} \ge 0$ and integers

where $a_i{}' = \sum_{j\in J} u_{ij} \forall i \in I, \text{a}'_{m+l} = \sum_{j\in J}\sum_{i\in I} u_{ij} - P = b'_{n+l} ; b_j{}' = \sum_{i\in I} u_{ij} \forall j \in J$

$$c'_{ij} = c_{ij}, \forall i \in I, j \in J, c'_{m+l,j} = c'_{i,n+l} = 0, \forall i \in I, j \in J, c_{m+1,n+1} = M$$

$$F'_i = F_i \forall i = (1,2,\ldots\ldots,\text{m}), \text{F}'_{m+1} = 0$$

$$I' = \{1,2,\ldots,m,m+1\}, J' = \{1,2,\ldots.,n,n+1\}$$

3 Theoretical development:

Definition: *Corner feasible solution:* A basic feasible solution $\{y_{ij}\}$ $i \in I', j \in J'$ to problem (P_2) is calleda corner feasible solution (cfs) if $y_{m+1,n+1} = 0$

Theorem 1: A non corner feasible solution of problem (P_2) cannot provide a basic feasible solution to problem (P_1).

Proof: Let $\{y_{ij}\}_{I'\times J'}$ be a non corner feasible solution to problem (P_2).Then, $y_{m+1,n+1} = \lambda (> 0)$ thus,

$$\sum_{i\in I'} y_{i,n+1} = \sum_{i\in I} y_{i,n+1} + y_{m+1,n+1}$$

$$= \sum_{i\in I} y_{i,n+1} + \lambda$$

$$= \sum_{i\in I} \sum_{j\in J} u_{ij} - P$$

Therefore, $\sum_{i\in I} y_{i,n+1} = \sum_{i\in I} \sum_{j\in J} u_{ij} - (\text{P} + \lambda)$

Now, for $i \in I$,

$$\sum_{j\in J'} y_{ij} = a_i = \sum_{j\in J} u_{ij}$$

$$\sum_{i\in I}\sum_{j\in J'} y_{ij} = \sum_{i\in i}\sum_{j\in J} u_{ij}$$

$$\sum_{i\in I}\sum_{j\in J} y_{ij} + \sum_{i\in I} y_{i,n+1} = \sum_{i\in I}\sum_{j\in J} u_{ij}$$

$$\sum_{i\in I}\sum_{j\in J} y_{ij} + \sum_{i\in I}\sum_{j\in J} u_{ij} - (\text{P}+\lambda) = \sum_{i\in I}\sum_{j\in J} u_{ij}$$

Therefore, $\sum_{i\in I}\sum_{j\in J} y_{ij} = P+\lambda$

This implies that total quantity transported from the sources in I to the destinations in J is $P + \lambda > P$, a contradiction to assumption that total flow is P and hence $\{y_{ij}\}_{I'\times J'}$ cannot provide a feasible solution to problem (P_1)

Lemma 1: There is one to one correspondence between a feasible solution of problem (P_2) and a corner feasible solution of problem (P_2').

Proof: Let $\{x_{ij}\}_{I\times J}$ be a feasible solution of problem (P_2).

So by relation (1), we have $x_{ij} \le u_{ij}$ which implies $\sum_{j\in J} x_{ij} \le \sum_{j\in J} u_{ij}$ (1.4)

By relation (1) and (1.4), we get

$$a_i \le \sum_{j\in J} x_{ij} \le \sum_{j\in J} u_{ij} = a_i'$$

Similarly, $b_j \le \sum_{i\in I} x_{ij} \le \sum_{i\in I} u_{ij} = b_j'$

Define $\{y_{ij}\}_{I'\times J'}$ by the following transformation

$$y_{ij} = x_{ij}, i\in I, j\in J \quad (1.5)$$

$$y_{i,n+1} = \sum_{j\in J} u_{ij} - \sum_{j\in J} x_{ij}; \forall i\in I \quad (1.6)$$

$$y_{m+1,j} = \sum_{i\in I} u_{ij} - \sum_{i\in I} x_{ij}; \forall j\in J \quad (1.7)$$

$$y_{m+1,n+1} = 0 \quad (1.8)$$

It can be shown that $\{y_{ij}\}$ so defined is a cfs to problem (P_2')

Relation (1) and (1.5) imply that $l_{ij} \le y_{ij} \le u_{ij}; \forall i\in I, j\in J$

Relation (1) and (1.6) imply that $0 \le y_{i,n+1} \le \sum_{j \in J} u_{ij} - a_i ; \forall \mathrm{i} \in \mathrm{I}$

Relation (1) and (1.7) imply that $0 \le y_{m+1,j} \le \sum_{i \in I} u_{ij} - b_j ; \forall j \in J$

Relation (1.8) implies that $y_{m+1,n+1} \ge 0$

Also for $i \in I$,relation (1.5) and (1.6) imply that

$$\sum_{j \in J'} y_{ij} = \sum_{j \in J} y_{ij} + y_{i,n+1} = \sum_{j \in J} x_{ij} + \sum_{j \in J} u_{ij} - \sum_{j \in J} x_{ij} = \sum_{j \in J} u_{ij} = a_i$$

For $i = m+1$

$$\sum_{j \in J'} y_{m=1,j} = \sum_{j \in J} y_{ij} + y_{m+1,n+1} = \sum_{j \in J} \left(\sum_{i \in I} u_{ij} - \sum_{i \in I} x_{ij} \right)$$

$$= \sum_{i \in I} \sum_{j \in J} u_{ij} - \sum_{i \in I} \sum_{j \in J} x_{ij}$$

$$= \sum_{i \in I} \sum_{j \in J} u_{ij} - P = a'_{m+1}$$

Therefore, $\sum_{j \in J'} y_{ij} = a'_i ; \forall i \in I'$

Similarly, it can be shown that $\sum_{i \in I} y_{ij} = b'_j ; \forall j \in J'$

Therefore, $\{y_{ij}\}_{I' \times J'}$ is a cfs to problem ($P_2^{'}$).

Conversely, let $\{y_{ij}\}_{I' \times J'}$ be a cfs to problem ($P_2^{'}$). Define x_{ij} $, i \in I, j \in J$ by the following transformation.

$$x_{ij} = y_{ij}, i \in I, j \in J \qquad (1.9)$$

It implies that $l_{ij} \le x_{ij} \le u_{ij}, i \in I, j \in J$

Now for $i \in I$,the source constraints in problem ($P_2^{'}$) imply

$$\sum_{j \in J'} y_{ij} = a_i^{'} = \sum_{j \in J} u_{ij}$$

$$\sum_{j \in J} y_{ij} + y_{i,n+1} = \sum_{j \in J} u_{ij}$$

$$\Rightarrow a_i \le \sum_{j \in J} y_{ij} \le \sum_{j \in J} u_{ij} \qquad (\text{ since } 0 \le y_{i,n+1} \le \sum_{j \in J} u_{ij} - a_i ; \forall i \in I \ )$$

Hence, $\sum_{j \in J} y_{ij} \ge a_i, i \in I$ and subsequently, $\sum_{j \in J} x_{ij} \ge a_i, i \in I$

Similarly, for $j \in J, \sum_{j \in J} y_{ij} \geq b_j ; \forall j \in J$ and subsequently, $\sum_{i \in I} x_{ij} \geq b_j ; \forall j \in J$

For $i = m+1$

$$\sum_{j \in J'} y_{m+1,j} = a'_{m+1} = \sum_{i \in I} \sum_{j \in J} u_{ij} - P$$

$$\Rightarrow \sum_{j \in J} y_{m+1,j} = \sum_{i \in I} \sum_{j \in J} u_{ij} - P \text{ because } y_{m+1,n+1} = 0 \qquad (1.10)$$

Now for $j \in J$ the destination constraints in problem (P_2') give

$$\sum_{i \in I} y_{ij} + y_{m+1,j} = \sum_{i \in I} u_{ij}$$

Therefore, $\sum_{i \in I} \sum_{j \in J} y_{ij} + \sum_{j \in J} y_{m+1,j} = \sum_{i \in I} \sum_{j \in J} u_{ij}$

By relation (1.10), we have

$$\sum_{i \in I} \sum_{j \in J} y_{ij} = \sum_{i \in I} \sum_{j \in J} u_{ij} - \sum_{j \in J} y_{m+1,j} = P$$

$$\Rightarrow \sum_{i \in I} \sum_{j \in J} x_{ij} = P$$

Therefore, $\{x_{ij}\}_{I \times J}$ is a feasible solution to problem (P_2).

Remark 1 : If problem (P_2') has cfs, then since $c'_{m+1,n+1} = M$ and $d'_{m+1,n+1} = M$,it follows that non corner feasible solution can not be an optimal solution to problem (P_2).

Lemma 2 : The value of the objective function of problem (P_2) at a feasible solution $\{x_{ij}\}_{I \times J}$ is equal to the value of the objective function of problem (P_2') at its corresponding cfs $\{y_{ij}\}_{I' \times J'}$ and conversely.

Proof: The value of the objective function of problem (P_2) at a feasible solution $\{x_{ij}\}_{I \times J}$ is

$$\min z = \left[\frac{\sum_{i \in I} \sum_{j \in J} c'_{ij} x_{ij}}{\sum_{i \in I} \sum_{j \in J} d'_{ij} x_{ij}} + \frac{\sum_{i \in I} F'_i}{\sum_{i \in I} G'_i} \right]$$

because $\left\{\begin{matrix} c'_{ij} = c_{ij}, \forall i \in I, j \in J \\ x_{ij} = y_{ij}, \forall i \in I, j \in J \\ c'_{i,n+1} = c'_{m+1,j} = c_{m+1,n+1} = 0 \\ F'_{m+1} = 0, F'_i = F_i, \forall i \in I \end{matrix}\right\}$

= objective function value of problem (P_2) at $\{x_{ij}\}$.Converse can be proved in a similar way.

Lemma 3: : There is a one to one correspondence between the optimal solution among the corner feasible solution to problem (P_2') and the optimal solution to problem(P_2).

Proof : Let $\{\hat{x}_{ij}\}_{I \times J}$ be an optimal solution to problem (P_2) with the value of objective function as Z^0 .Since $\{\hat{x}_{ij}\}_{I \times J}$ is an optimal solution, $\therefore$ $\{x_{ij}\}$ is a feasible solution to problem (P_2) . Then by lemma 1, there exist a corresponding feasible solution $\{\hat{y}_{ij}\}_{I' \times J'}$ is Z^0 [refer to lemma 2]

we will show that $\{\hat{y}_{ij}\}_{I' \times J'}$ is the optimal solution to problem ($P_2^{'}$).

Now,Let if possible, $\{\hat{y}_{ij}\}$ be not an optimal solution to problem ($P_2^{'}$).Therefore there exist a feasible solution $\{y'_{ij}\}$ say to problem ($P_2^{'}$) having the value of objective function $Z' < Z^0$ .Let $\{x'_{ij}\}$ be the corresponding feasible solution to problem (P_2). Then by theorem 2,

$$Z' = \left[\frac{\sum_{i \in I} \sum_{j \in J} c'_{ij} x_{ij}}{\sum_{i \in I} \sum_{j \in J} d'_{ij} x_{ij}} + \frac{\sum_{i \in I} F'_i}{\sum_{i \in I} G'_i} \right]$$

Which contradicts that $\{\hat{x}_{ij}\}$ is an optimal solution to problem (P_2).

Similarly, starting from an optimal feasible solution to problem ($P_2^{'}$), one can derive an optimal corner feasible solution to problem (P_2) having the same objective function value.

Theorem 2: Optimizing problem ($P_2^{'}$) is equivalent to optimizing problem (P_2) provided problem (P_2) has a feasible solution.

Proof: As problem (P_2) has a feasible solution, by lemma 1, there exists a cfs to problem ($P_2^{'}$).Thus by remark 1, an optimal solution to problem ($P_2^{'}$) will be a cfs. Hence, by lemma 3 , an optimal solution to problem (P_2) can be obtained.

4 Algorithm

Step 1.starting from the given non linear capacitated transportation problem (P_1) with enhanced flow, form a related transportation problem (P_2) by introducing a dummy source and a dummy destination with

$$a_i^{'} = \sum_{j\in J} u_{ij}; \forall i \in I \ , \ a^{'}_{m+1} = \sum_{i\in I}\sum_{j\in J} u_{ij} - P = b^{'}_{n+1} \ , \ b_j^{'} = \sum_{i\in I} u_{ij}; \forall j \in J \ ,$$

$$c^{'}_{ij} = c_{ij} \forall i \in I, j \in J \ , \ c^{'}_{m+1,j} = c^{'}_{i,n+1} = 0; \forall i \in I, j \in J \ , \ c^{'}_{m+1,n+1} = M$$

Step 2 : Find an initial basic feasible solutionto (P_2) with respect to variable cost only. Let B be its corresponding basis.

Step 3 : Calculate the fixed cost of the current basic feasible solution and denote it by F(current), where F(current) $= \sum_{i=1}^{m} F_i$

Step 4(a) : Find $\Delta F_{ij} = F(\text{NB}) - \text{F(current)}$ where F(NB) is the total fixed cost obtained when some non basic cell (i, j) undergoes change.

Step 4(b) : Calculate $\theta_{ij}, (\text{c}_{ij} - \text{z}_{ij})$ for all non basic cells such that

$$u_i + v_j = c_{ij}; \forall (\text{i, j}) \in \text{B}$$
$$\text{u}_i + v_j = z_{ij}; \forall (\text{i, j}) \in \text{N}_1 \& N_2$$

Θ_{ij} = level at which a non basic cell (I,j) enters the basis replacing some basic cell of B.

N_1 and N_2 denotes the set of non basic cells (I,j) which are at their lower and upper bounds respectively.

Note : u_i, v_j are the dual variables which are determined by using above equations and taking one of the u_i 's or v_j 's as zero.

Step 4(c) : Find $R^1_{ij}; \forall (\text{i, j}) \in \text{N}_1$ and $R^2_{ij}; \forall (\text{i, j}) \in \text{N}_2$ where $R^1_{ij} = \theta_{ij}(\text{c}_{ij} - \text{z}_{ij}) + \Delta \text{F}_{ij} \geq 0; \forall (\text{i, j}) \in \text{N}_1$ and $R^2_{ij} = -\theta_{ij}(\text{c}_{ij} - \text{z}_{ij}) + \Delta \text{F}_{ij} \geq 0; \forall (\text{i, j}) \in \text{N}_2$

Step 5 : If $R^1_{ij} \geq 0; \forall (\mathrm{i, j}) \in \mathrm{N}_1$ and $R^2_{ij} \geq 0; \forall (\mathrm{i, j}) \in \mathrm{N}_2$ then the current solution so obtained is the optimal solution to $(P_2^{'})$,Go to step 5. Otherwise some (I,j) ∈ N_1 for which $R^1_{ij} < 0$ or some (I,j) ∈ N_2 for which $R^2_{ij} < 0$ will undergo change. Go to step 3.

Step 6 : Let Z^1 be the optimal cost of $(P_2^{'})$ yielded by the basic feasible solution $\{y'_{ij}\}$ .Find all alternate solutions to the problem $(P_2^{'})$ with the same value of objective function. Let these solutions be $X_1, X_2, \ldots\ldots, X_n$ and $T^1 = \min_{X_1, X_2, \ldots\ldots, X_n} \{ \max_{i \in I', j \in J'} (t_{ij} / x_{ij} > 0) \}$. Then the corresponding pair (Z^1, T^1) will be the first time cost trade off pair for the problem (P_1).To find the second cost-time trade off pair, go to step 7.

Step 7 : Define $c^1_{ij} = \begin{Bmatrix} M & if t_{ij} \geq T^1 \\ c_{ij} & if t_{ij} < T^1 \end{Bmatrix}$ where M is a sufficiently large positive number .From the corresponding capacitated fixed charge transportation problem with variable cost c^1_{ij} .Repeat the above process till the problem becomes infeasible. The complete set of time cost trade off pairs of (P_1) at the end of q^{th} iteration are given by $(Z^1, T^1), (Z^2, T^2), \ldots\ldots\ldots\ldots (Z^q, T^q)$ where $Z^1 \leq Z^2 \leq \ldots\ldots\ldots \leq Z^q$ and $T^1 > T^2 > \ldots\ldots > T^q$.

Remark : The pair (Z^1, T^q) with minimum cost and minimum time is the ideal pair which cannot be achieved in practice except in some trivial case.

Convergence of the algorithm: The algorithm will converge after a finite number of steps because we are moving from one extreme point to another extreme point and the problem becomes infeasible after a finite number of steps.

5. Numerical Illustration:

Consider a 3*4 non linear capacitated transportation problem with specified flow

Table 1

	D1	D2	D3	D4	ai
O1	5 1	9 2	9 4	8 7	10
O2	4	6	2	5	6
O3	4	1	2	3	8
rj	2	3	4	6	

Table2: Anoptimalsolutionofproblem(P2)

	D1	D2	D3	D4	D5	D6	ai	u^1_i	u^2_i
O1	5 **2** 1	9 **3** 2	9 4	8 7	5 **5** 1	0 **0** 0	10	0	0
O2	4 3	6 7	2 **4** 4	5 6	2 4	0 **2** 0	6	0	0
O3	4 2	1 9	2 5	3 **6** 2	3 2	0 **2** 0	8	0	0
' bj	2	3	4	6	5	4			
1	5	9	2	3	5	0			
2	1	2	4	2	1	0			

Note: Entries in bold are basic cells.

Here $z_1 = 88$, $z_2 = 41$.

Table3:Calculationofoptimalitycondition

NB	O1D3	O1D4	O2D1	O2D2	O2D4	O2D5	O3D1	O3D2	O3D3	O3D5
华ij	0	0	2	2	2	2	2	2	2	2
1	7	5	-1	-3	2	-3	-1	-8	0	-2
2	0	5	2	5	4	3	1	7	1	1
Rij	0	0	262	574	900	246	90	352	176	4

Table4:Optimalsolutionofproblem(P1)

	D1	D2	D3	D4	ai
O1	5 **7**	9 **3**	9	8	10
O2	4	6	2 **4**	5	6
O3	4	1	2	3 **6**	8
rj	2	3	4	6	

$z_1 = 88$, $z_2 = 41$.Therefore minimum cost = Z = (88x41) = 3608 for problem (P1)

Conclusion:

In order to solve a transportation problem with specified flow, a related transportation problem is formed. It is shown that optimal solution to the specified problem may be obtained from the optimal solution of the related transportation problem. Since optimal solution of the related transportation problem is attain a bleat an extreme point, therefore optimal basic feasible solution of the related problem will give an optimal solution of the given problem

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