International Journal of Engineering & Scientific Research (IJMRA Publications)
Vol. 6 Issue 4, April 2018,
ISSN: 2347-6532 Impact Factor: 6.660
Journal Homepage: http://www.esrjournal.com Email: esrjeditor@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Multi Item Inventory Model with Uncertain Lead Time and Varying Holding Cost via Geometric Programming; A Fuzzy Approach

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Abstract: In this paper, a multi item objective function fuzzy inventory model has been derived with uncertain lead time and decreasing holding cost using the geometric programming method. Here lead time is allowed and it is considered to be a fuzzy number. The decreasing holding cost is considered to be a continuous function of a product quantity. The total cost is minimized under the restrictions of the limitation on the total number of orders and the limitation on the total holding cost using geometric programming. Finally, a numerical example and sensitivity analysis in fuzzy environment is given to illustrate this model. Graded mean representation method is used to defuzzify the results.

Keywords: Lead time, Varying Holding Cost, Trapezoidal fuzzy numbers, Graded Mean Representation method, Geometric Programming.

Article Received: 3rd March, 2018 Article Revised: 14th March, 2018 Article Accepted: 24th March, 2018

1. Introduction

Inventories are a major module of any logistics system and as such require be scheduled, supervision and scheming in order to achieve the basic aims of minimizing costs at satisfactory levels of investment and providing the preferred levels of customer service. Inventory management needs to recognize it cannot work in isolation within the logistics system. In the enlargement of an inventory system to achieve the two basic serviceable objectives related to investment cost and service level.

Inventory management is widely looked upon as a day to day process concerned with meeting specified short term targets. It does, however, have a major role to play in the short, medium and long term developments of an organization and should be an integral part of the business forecasting cycle.

The lead time is defined as the interval between deciding that an order needs to be placed and the order being physically available for issue. This should not be confused with supplier delivery time, which will cover a shorter period, but does not include the administrative processes prior to and following the delivery time as well as the physical activity of receiving and storing the stock. The only lead time variations that need to be accounted for, as far as inventory control is concerned, are the unexpected variations, typically caused by an increase in the supply delay due to a stock-out experienced by the supplier itself.

Geometric programming provides an influential tool for solving a variety of engineering optimization problems. Geometric programming is a technique developed for solving algebraic nonlinear programming problems subject to linear or nonlinear constraints, is useful in the study of a variety of optimization problems. Efficient algorithms have been developed for solving the geometric programming problems when the cost and the constraint coefficients are known exactly. A geometric program (GP) is a type of mathematical optimization problem characterized by objective and constraint functions that have a unique form. Recently developed solution methods can solve even all-encompassing GPs extremely efficient and reliable.

Abou-EL-Ata and Kotb [1] developed that multi-item EOQ inventory model with varying holding cost under two restrictions: A geometric programming approach. Abou-el-ata, Fergany and El-Wakeel

[2] opined that probabilistic multi-item inventory model with varying order cost under two restrictions: a geometric programming approach. Duffin, Peterson and Zener [3] developed that geometric programming –theory and application. Das, Roy and Maiti [4] derived that multi-item inventory model with quantity dependent inventory costs and demand-dependent unit cost under imprecise objective and restrictions: a geometric programming approach. Gupta, [5]
Hariri and Abou-el-ata [6] explained that multi-item production lot-size inventory model with varying order cost under a restriction: a geometric programming approach. Jung and Klein [7] constructed that optimal inventory policy under decreasing cost functions via Geometric Programming. Kotb and Fergany [8] asserted that multi-item EOQ Model with varying holding: a geometric programming approach. Kotb and Fergany [9] discussed that multi-item EOQ model with both demand-dependent unit cost and varying leading time via geometric programming. Maloney and Klein [10] established that constrained multi-item inventory systems**: a**n implicit approach. Mandal, Roy and Maiti [11] formulated that inventory model of deteriorated items with a constraint: a geometric programming approach. Mandal, Roy and Maiti [12] discussed that multi-objective fuzzy inventory model with three constraints: a geometric programming approach. Ojha and Biswal [13] derived that multi-objective geometric programming problem with weighted mean method. Werners [14] developed that interactive multiple objective programming subject to flexible constraints. Worral and Hall [15] explained that the analysis of an inventory control model using posynomial geometric programming. Zadeh [16] constructed that Fuzzy sets. Zimmermann [17] opined that description and optimization of fuzzy systems.

This paper deals an uncomplicated and realistic situation and obtains the optimal solution in multi item inventory model with uncertain lead time and varying holding cost using geometric programming through fuzzy.

We focus here on one important modified method (i.e) geometric programming method. Based on this modified method we derive an optimal total annual cost. Lead time is taken as a trapezoidal fuzzy number and for defuzzification we use graded mean representation method.

2. Methodology

2.1 Fuzzy Numbers

Any fuzzy subset of the real line R, whose membership function μ_A satisfied the following conditions, is a generalized fuzzy number $\tilde{A}$.

(i) μ_A is a continuous mapping from R to the closed interval [0, 1].

(ii) $\mu_A = 0, -\infty < x \le a_1,$

(iii) $\mu_A =$ L(x) is strictly increasing on $[a_1, a_2]$

(iv) $\mu_A = w_A, a_2 \le x \le a_3$

(v) $\mu_A =$ R(x) is strictly decreasing on $[a_3, a_4]$

(vi) $\mu_A = 0, a_4 \le x < \infty$

where $0 < w_A \le 1$ and a_1, a_2, a_3 and a_4 are real numbers. Also this type of generalized fuzzy number be denoted as $\tilde{A} = (a_1, a_2, a_3, a_4 : w_A)_{LR}$; When $w_A = 1$, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$

Trapezoidal Fuzzy Number

A trapezoidal fuzzy number $\tilde{A}$ = (a, b, c, d) is represented with membership function $\mu_{\tilde{A}}$ as:

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \frac{x-a}{b-a}, & when\ a \le x \le b; \\ 1, & when\ b \le x \le c; \\ R(x) = \frac{d-x}{d-c}, & when\ c \le x \le d; \\ 0, & otherwise \end{cases}$$

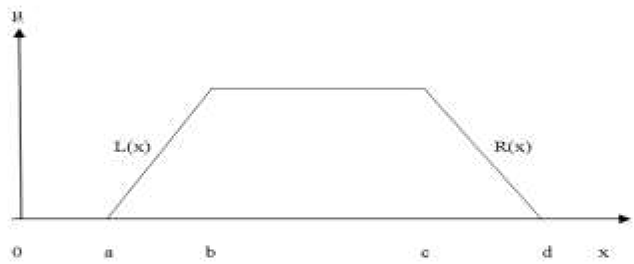

Fig.1: Trapezoidal Fuzzy Number

2.3 The Function Principle

The function principle is used for the operation of addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ *and* $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers, then arithmetical operations are defined as:

1. $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
2. $\tilde{A} \otimes \tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4)$
3. $\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
4. $\tilde{A} \oslash \tilde{B} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$
5. $\alpha \otimes \tilde{A} = \begin{cases} (\alpha a1, \alpha a2, \alpha a3, \alpha a4), & \alpha \ge 0 \\ (\alpha a4, \alpha a3, \alpha a2, \alpha a1), & \alpha < 0 \end{cases}$

2.4 Geometric Programming

2.4.1 Formulation of Multi-Objective Geometric Programming

A multi-objective geometric programming problem can be defined as:

Find $x = (x_1, x_2, ..., x_n)^T$

so as to

$$\min g_{k0}(x) = \sum_{i=1}^{T_{k0}} C_{k0p} \prod_{j=1}^{n} x_j^{b_{k0pj}},\ k = 1,2,...,l$$

subject to the constraints

$$g_i(x) = \sum_{p=1}^{T_i} C_{ip} \prod_{j=1}^{n} x_j^{d_{ipj}} \le 1,\ i = 1,2,...,m$$

$$x_j > 0,\ j = 1,2,...,n$$

where C_{k0p}, $\forall$ k and p are positive real numbers and d_{ipj} and a_{k0pj} are real numbers for all i, j, k, p.

Here l = number of minimization type objective functions

m = number of inequality type constraints and
n = number of strictly positive decision variables.

2.4.2 Weighted Method of Multi-Objective Geometric Programming

The weighted method is the easiest and functional multi-objective optimization which has been extensively applied to attain the optimal solution of the multi - objective function within the convex objective space.

According to the number of objective functions, the weights $w_1, w_2, ..., w_l$ are assigned to define a new minimization type objective function Z (w) which can be defined as

$$\min Z(x) = \sum_{k=1}^{l} w_k\, g_{k0}(x)$$

$$= \sum_{k=l}^{l} w_k \left(\sum_{p=1}^{T_{k0}} C_{k0p} \prod_{j=1}^{n} x_j^{b_{k0pj}} \right)$$

$$= \sum_{k=1}^{l} \sum_{i=1}^{T_{k0}} w_k\, C_{k0p} \prod_{j=1}^{n} x_j^{b_{k0pj}}$$

Subject to the constraints

$$\sum_{p=1}^{T_i} C_{ip} \prod_{j=1}^{n} x_j^{d_{ipj}} \leq 1,\ i = 1,2,...,m$$

$$x_j > 0,\ j = 1,2,...,n$$

Where $\sum_{k=1}^{l_i} w_k = 1,\ w_k > 0,\ k = 1,2,...,l$

2.4.3 Dual Form of Geometric Programming

According to Duffin, Peterson and Zener [3], the given model is transformed to the corresponding dual form of the geometric programming is

$$\max_{w} = \prod_{i=1}^{T_0} \left(\frac{w_k\, C_{k0p}}{w_{0p}} \right)^{w_{0p}} \prod_{i=1}^{m} \prod_{p=1}^{T_i} \left(\frac{w_{i0}\, C_{ip}}{w_{ip}} \right)^{w_{ip}} \prod_{p=1}^{T_i} \lambda(w_{ip})^{\lambda(w_{ip})}$$

Subject to the constraints

$$\sum_{p=1}^{T_0} w_{0p} = 1$$

$$\sum_{i=1}^{m} \sum_{p=1}^{T_i} b_{ipj}\, w_{ip} + \sum_{i=1}^{m} \sum_{p=1}^{T_i} d_{ipj}\, w_{ip} = 0,\ \ j = 1,2,...,n$$

$$w_{ip} \geq 0,\ \forall\, p, j$$

Where $\sum_{k=1}^{l_i} w_k = 1,\ w_k > 0,\ k = 1,2,...,l$

2.5 Graded Mean Integration Representation Method

It $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number, then the graded mean integration representation of $\tilde{A}$ being,

$$p(\tilde{A}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

2.6 Notations and Assumptions

The mathematical model in this paper is developed on the basis of the following notations and assumptions.

2.6.1 Notations

For i^{th} item (i = 1, 2, 3, …, n)

n	-	number of items
q_i	-	production quantity batch (decision variable) for the i^{th} item
R_i	-	annual demand rate for the i^{th} item
r_i	-	annual rate of production for the i^{th} item
P_i	-	unit purchase or production cost
C_i	-	unit holding cost per item
S_i	-	set up or ordering cost per order
$C_i(q_i)$	-	varying holding cost for the i^{th} item
M_1	-	limitation on the total number of orders
M_2	-	limitation on the total holding cost
$\tilde{L}_i$	-	fuzzy lead time
$T\tilde{C}(q_i)$	-	fuzzy total annual relevant cost

2.6.2 Assumptions

The following basic assumptions about the mathematical model are developed in our inventory problem.

- Demand rate is uniform over time.
- Shortages are not allowed.
- Lead time is allowed and taken as a trapezoidal fuzzy number.
- The rate of production for each product is finite and constant.
- Transportation cost is not considered.
- The holding cost for the i^{th} item is a decreasing continuous function of the production quantity q_i and we are taken in the form is $C_i(q_i) = \alpha + \frac{\beta}{q_i}$,

  i = 1, 2,…, n, $\alpha > 0$ and $\beta > 0$ are real constants preferred to provide the best robust of the anticipated cost function, since $C_i(q_i)$ must be non-negative.
- Our aim is to minimize the annual pertinent total cost.

3. Model Formulation

3.1 Proposed Inventory Model in Fuzzy Sense

From the above notations and assumptions, we obtain the total annual relevant cost for the proposed inventory model in fuzzy environment.

Here, Lead time (L_i) is taken as a trapezoidal fuzzy number.

Let $\tilde{L}_i$ - fuzzy lead time

Then, the fuzzy total annual relevant cost of the EOQ model is given by,

$T\tilde{C}(q_i)$ = production cost + setup cost + holding cost + lead time

$$T\tilde{C}(q_i) = \sum_{i=1}^{n} \left\{ R_i P_i + \frac{R_i S_i}{q_i} + \frac{q_i}{2}\left(1 - \frac{R_i}{r_i}\right) C_i(q_i) + \tilde{L}_i \right\} \tag{1}$$

Substituting $C_i(q_i)$ in equation (1), yields,

$$\tilde{TC}(q_i) = \sum_{i=1}^{n}\left\{R_i\,P_i+\tilde{L}_i\right\} + \sum_{i=1}^{n}\left\{\frac{R_i\,S_i}{q_i}+\frac{q_i}{2}\left(1-\frac{R_i}{r_i}\right)\left(\alpha+\beta\,q_i^{-1}\right)\right\} \quad (2)$$

$$\tilde{TC}(q_i) = \sum_{i=1}^{n}\left\{R_i\,P_i+\tilde{L}_i+\frac{\beta}{2}\left(1-\frac{R_i}{r_i}\right)\right\} + \sum_{i=1}^{n}\left\{\frac{R_i\,S_i}{q_i}+\frac{\alpha\,q_i}{2}\left(1-\frac{R_i}{r_i}\right)\right\} \quad (3)$$

There is a limitation on the total number of orders and a limitation on the total holding cost and these limitations are the constraints.
The constraints are,

$$\sum_{i=1}^{n}\frac{R_i}{q_i}\le M_1 \quad \text{and} \quad \sum_{i=1}^{n}\frac{q_i}{2}\left(1-\frac{R_i}{r_i}\right)C_i\le M_2 \quad (4)$$

where M_1 and M_2 are the limits on total number of orders and total holding cost respectively.

The term $\sum_{i=1}^{n}\left\{R_i\,P_i+\tilde{L}_i+\frac{\beta}{2}\left(1-\frac{R_i}{r_i}\right)\right\}$ is constant and hence we omit that term.

To solve this primal function which is a convex programming problem, let us write it in the subsequent simplified version of equation (3), then,

$$\min TC=\sum_{i=1}^{n}\left\{\frac{R_i\,S_i}{q_i}+\frac{\alpha R_i'\,q_i}{2}\right\} \quad (5)$$

Subject to the constraints

$$\sum_{i=1}^{n}\frac{R_i}{M_1\,q_i}\le 1 \quad \text{and} \quad \sum_{i=1}^{n}\frac{C_i\,q_i\,R_i'}{2M_2}\le 1 \quad \text{where } R_i'=1-\frac{R_i}{r_i} \quad (6)$$

Apply the geometric programming technique to equations (5) and (6), we have the enlarged predual function might be written in the following form as,

$$G(\underline{W})=\prod_{i=1}^{n}\left(\frac{R_i\,S_i}{W_{1i}}\right)^{W_{1i}}\left(\frac{\alpha\,R_i'}{2W_{2i}}\right)^{W_{2i}}\left(\frac{R_i}{M_1\,W_{3i}}\right)^{W_{3i}}\left(\frac{C_i\,R_i'}{2M_2\,W_{4i}}\right)^{W_{4i}} q_i^{-W_{1i}+W_{2i}-W_{3i}+W_{4i}} \quad (7)$$

where the dual vector $(\underline{W})=W_{ji}\,, 0<W_{ji}<1$, j = 1,2,3,4, i = 1,2,3,…,n is arbitrary and for convenience we choose the normality condition is,

$$W_{1i}+W_{2i}=1 \quad (8)$$

We have to choose $(\underline{W})$ such that the exponent of q_i is zero, we make the R.H.S of equation (7) is independent of the decision variable.
So, we choose the orthogonality condition is,

$$-W_{1i}+W_{2i}-W_{3i}+W_{4i}=0 \quad (9)$$

Then again, the problem is to select the minimum solution of the weights W_{ji}^* .

Solve equations (8) and (9), we get,

$$W_{1i}=\frac{1-W_{3i}+W_{4i}}{2} \quad \text{and} \quad W_{2i}=\frac{1+W_{3i}-W_{4i}}{2} \quad (10)$$

Substitute the value of W_{1i} and W_{2i} in equation (7), then the dual function is given by,

$$g(W_{3i},W_{4i})=\prod_{i=1}^{n}\left(\frac{2R_iS_i}{1-W_{3i}+W_{4i}}\right)^{\left(\frac{1-W_{3i}+W_{4i}}{2}\right)}\left(\frac{\alpha R_i'}{1+W_{3i}-W_{4i}}\right)^{\left(\frac{1+W_{3i}-W_{4i}}{2}\right)}\left(\frac{R_i}{M_1W_{3i}}\right)^{W_{3i}}\left(\frac{C_iR_i'}{2M_2W_{4i}}\right)^{W_{4i}} \quad (11)$$

Taking ‘log’ on both sides of equation (11), we get,

$$\log\left[g\left(W_{3i},W_{4i}\right)\right]=\left(\frac{1-W_{3i}+W_{4i}}{2}\right)\log\left(2R_iS_i\right)-\left(\frac{1-W_{3i}+W_{4i}}{2}\right)\log\left(1-W_{3i}+W_{4i}\right)+$$
$$\left(\frac{1+W_{3i}-W_{4i}}{2}\right)\log\left(\alpha R_i^{'}\right)-\left(\frac{1+W_{3i}-W_{4i}}{2}\right)\log\left(1+W_{3i}-W_{4i}\right)+$$
$$W_{3i}\log\left(R_i\right)-W_{3i}\log\left(M_1W_{3i}\right)+W_{4i}\log\left(C_iR_i^{'}\right)-W_{4i}\log\left(2M_2W_{4i}\right) \quad (12)$$

Differentiate equation (12) partially w.r.to W_{3i}, we get,

$$\frac{\partial\log\left[g\left(W_{3i},W_{4i}\right)\right]}{\partial W_{3i}}=\frac{1}{2}\left[\log\left(\frac{\alpha R_i^{'}}{2R_iS_i}\right)\left(\frac{R_i}{M_1e}\right)^2\left(\frac{1-W_{3i}+W_{4i}}{1+W_{3i}-W_{4i}}\right)\left(\frac{1}{W_{3i}^2}\right)\right] \quad (13)$$

Set the equation (13) to zero and simplifies, we get,

$$\left(\frac{\alpha R_i^{'}}{2R_iS_i}\right)\left(\frac{R_i}{M_1e}\right)^2\left(\frac{1-W_{3i}+W_{4i}}{1+W_{3i}-W_{4i}}\right)\left(\frac{1}{W_{3i}^2}\right)=1 \quad (14)$$

Differentiate equation (12) partially w.r.to W_{4i}, we get,

$$\frac{\partial\log\left[g\left(W_{3i},W_{4i}\right)\right]}{\partial W_{4i}}=\frac{1}{2}\left[\log\left(\frac{2R_iS_i}{\alpha R_i^{'}}\right)\left(\frac{C_iR_i^{'}}{2M_2e}\right)^2\left(\frac{1+W_{3i}-W_{4i}}{1-W_{3i}+W_{4i}}\right)\left(\frac{1}{W_{4i}^2}\right)\right] \quad (15)$$

Set the equation (15) to zero and simplifies, we get,

$$\left(\frac{2R_iS_i}{\alpha R_i^{'}}\right)\left(\frac{C_iR_i^{'}}{2M_2e}\right)^2\left(\frac{1+W_{3i}-W_{4i}}{1-W_{3i}+W_{4i}}\right)\left(\frac{1}{W_{4i}^2}\right)=1 \quad (16)$$

Multiplying relation (14) by the relation (16), we have

$$W_{3i}W_{4i} = \frac{R_iR_i^1C_i}{2M_1M_2e^2} \quad (17)$$

From equation (17), we get the value of W_{3i} and W_{4i}

$$W_{3i} = \frac{R_iR_i^1C_i}{2M_1M_2e^2W_{4i}} \quad \text{and}$$

$$W_{4i} = \frac{R_iR_i^1C_i}{2M_1M_2e^2W_{3i}} \quad (18)$$

Substitute the value of W_{4i} in the equation (14), we get,

$$f\left(W_{3i}\right)=W_{3i}^4+W_{3i}^3-\left[\left(\frac{R_iR_i^{'}C_i}{2M_1M_2e^2}\right)-\left(\frac{\alpha R_i^{'}}{2R_iS_i}\right)\left(\frac{R_i}{M_1e}\right)^2\right]W_{3i}^2-\left(\frac{\alpha R_i^{'}}{2R_iS_i}\right)\left(\frac{R_i}{M_1e}\right)^2W_{3i}$$
$$-\left(\frac{R_iR_i^{'}C_i}{2M_1M_2e^2}\right)\left(\frac{\alpha R_i^{'}}{2R_iS_i}\right)\left(\frac{R_i}{M_1e}\right)^2 = 0$$

$$\Rightarrow f\left(W_{3i}\right)=W_{3i}^4+W_{3i}^3-\left(A_i-B_{3i}\right)W_{3i}^2-B_{3i}W_{3i}-A_iB_{3i}=0 \quad (19)$$

where $A_i=\left(\frac{R_iR_i^{'}C_i}{2M_1M_2e^2}\right)$ and $B_{3i}=\left(\frac{\alpha R_i^{'}}{2R_iS_i}\right)\left(\frac{R_i}{M_1e}\right)^2$

Substitute the value of W_{3i} in the equation (16), we get,

$$f(W_{4i}) = W_{4i}^4 + W_{4i}^3 - \left[\left(\frac{R_i R_i' C_i}{2M_1 M_2 e^2}\right) - \left(\frac{2R_i S_i}{\alpha R_i'}\right)\left(\frac{R_i' C_i}{2M_2 e}\right)^2\right] W_{4i}^2 - \left(\frac{2R_i S_i}{\alpha R_i'}\right)\left(\frac{R_i' C_i}{2M_2 e}\right)^2 W_{4i}$$

$$-\left(\frac{R_i R_i' C_i}{2M_1 M_2 e^2}\right)\left(\frac{2R_i S_i}{\alpha R_i'}\right)\left(\frac{R_i' C_i}{2M_2 e}\right)^2 = 0$$

$$\Rightarrow f(W_{4i}) = W_{4i}^4 + W_{4i}^3 - (A_i - B_{4i}) W_{4i}^2 - B_{4i} W_{4i} - A_i B_{4i} = 0 \qquad (20)$$

where $A_i = \left(\frac{R_i R_i' C_i}{2M_1 M_2 e^2}\right)$ and $B_{4i} = \left(\frac{2R_i S_i}{\alpha R_i'}\right)\left(\frac{R_i' C_i}{2M_2 e}\right)^2$

Combining equation (19) and (20), we get,

$$\Rightarrow f(W_{ji}) = W_{ji}^4 + W_{ji}^3 - (A_i - B_{ji}) W_{ji}^2 - B_{ji} W_{ji} - A_i B_{ji} = 0 \quad j = 3,4 \qquad (21)$$

where

$$A_i = \left(\frac{R_i R_i' C_i}{2M_1 M_2 e^2}\right),\ B_{3i} = \left(\frac{\alpha R_i'}{2R_i S_i}\right)\left(\frac{R_i}{M_1 e}\right)^2 \text{ and } B_{4i} = \left(\frac{2R_i S_i}{\alpha R_i'}\right)\left(\frac{R_i' C_i}{2M_2 e}\right)^2$$

Now,

$$f(0) = -A_i B_{ji} < 0,\ j = 3,4$$

$$f(1) = 2 - A_i (1 + B_{ji}) > 0,\ j = 3,4$$

Therefore, a root lies between 0 and 1 and j = 3, 4.

For calculation, we use the trial and error method.

We shall first verify that any root W_{ji}^*, j = 3,4 calculated from equation (21) maximizes $g(W_{ji}^*)$, j = 3, 4.

This is simply possible by finding the second derivative of the equations (13) and (15).

$$\frac{\partial^2 \log(W_{3i}, W_{4i})}{\partial W_{3i}^2} < 0,\ \frac{\partial^2 \log(W_{3i}, W_{4i})}{\partial W_{4i}^2} < 0 \text{ and } \frac{\partial^2 \log(W_{3i}, W_{4i})}{\partial W_{3i} \partial W_{4i}} > 0.$$

Hence, $\Delta = \left(\frac{\partial^2 \log(W_{3i}, W_{4i})}{\partial W_{3i} \partial W_{4i}}\right)^2 - \left(\frac{\partial^2 \log(W_{3i}, W_{4i})}{\partial W_{3i}^2}\right)\left(\frac{\partial^2 \log(W_{3i}, W_{4i})}{\partial W_{4i}^2}\right) < 0.$

Thus, the roots W_{3i}^* and W_{4i}^* are calculated from equation (21) maximize the dual function g(W_{3i}, W_{4i}).

Hence the minimum solution is W_{ji}^*, j = 1,2,3,4, where W_{3i}^*, W_{4i}^* are the solutions of equation (21) and W_{1i}^*, W_{2i}^* are calculated by substituting the values of W_{3i}^* and W_{4i}^* in the equation (10).

To calculate the optimal order quantity q_i^*, we apply Duffin and Peterson's theorem of geometric programming as:

$$\frac{R_i S_i}{q_i^*} = W_{1i}^* \, g\left(W_{1i}^*, W_{2i}^*\right) \text{ and}$$

$$\frac{\alpha R_i^{'} q_i^*}{2} = W_{2i}^* \, g\left(W_{1i}^*, W_{2i}^*\right)$$

Solving the above relations, then we obtain the optimal order quantity is,

$$\Rightarrow q_i^* = \sqrt{\frac{2 R_i S_i W_{2i}^*}{\alpha R_i^{'} W_{1i}^*}} \text{ , i = 1, 2, 3, …, n} \qquad (22)$$

Substitute equation (22) in equation (3), we get,

$$\min T\tilde{C} = \sum_{i=1}^{n}\left[R_i\, P_i + \frac{\beta\, R_i^{'}}{2} + \tilde{L}_i + \sqrt{\frac{\alpha\, R_i\, R_i^{'}\, S_i}{2 W_{1i}^* W_{2i}^*}} \right] \qquad (23)$$

4. Numerical Example

4.1 Numerical Example in Fuzzy Sense

The annual demand rate of an item is 200 units, the annual rate of production of an item is 500 units, unit purchase or production cost per item is Rs. 20, set up or ordering cost per order is Rs. 300 and the lead time is (0.1, 0.2, 0.3, 0.4). Assume that the total number of orders for each year is 1500 and the total holding cost is Rs.1000, α = Rs. 1 and β = 0.5. The order quantity and minimum total annual cost are to be determined.

Sol:

n = 1
R = 200 units
P = Rs. 20
r = 500 units
S = Rs. 300 / unit
L = (0.1, 0.2, 0.3, 0.4)
M_1 = 1500 ft^2
M_2 = Rs. 1000
α = Rs. 1 / unit
β = 0.5

Order Quantity

$$q^* = \left[\frac{2\, R\, S\, W_2^*}{\alpha\, R^{'}\, W_1^*}\right]^{1/2}$$

$$q^* = 436$$

Fuzzy minimum annual total relevant cost

$$\min T\tilde{C} = RP + \frac{\beta\, R^{'}}{2} + \tilde{L} + \sqrt{\frac{\alpha\, R\, R^{'}\, S}{2 W_1^* W_2^*}}$$

$\min T\tilde{C}$ = Rs. (4600.25, 4600.35, 4600.45, 4600.55)

Graded Mean Representation Method

$$P\left(T\tilde{C}\right) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

$P(\tilde{TC})$ = Rs. 4600.40

5. Sensitivity Analysis

A sensitivity analysis of this fuzzy model has been derived for varying β value. Here we consider an inventory model for three items with the following input data.

Table: 1

n	R (units)	r (units)	P (Rs.)	S (Rs.)	$\tilde{L}$	α (Rs.)
1	200	500	20	300	(0.1,0.2,0.3,0.4)	1
2	150	400	15	200	(0.1,0.2,0.3,0.4)	1
3	100	300	10	100	(0.1,0.2,0.3,0.4)	1

Assume that the total number of order for each year is 1500 ft^2 and the total holding cost is Rs.1,000

Table: 2

S.No	β	q_1^*	q_2^*	q_3^*	$C_1(q_1^*)$	$C_2(q_2^*)$	$C_3(q_3^*)$	min($\tilde{TC}$)
1	0.1	436	302	170	1.0002	1.0003	1.0006	7828.29
2	0.5	436	302	170	1.0012	1.0017	1.0029	7828.68
3	1	436	302	170	1.0023	1.0033	1.0059	7829.14
4	2	436	302	170	1.0046	1.0066	1.0118	7830.10
5	5	436	302	170	1.0115	1.0166	1.0294	7832.93
6	10	436	302	170	1.0229	1.0331	1.0588	7837.66
7	50	436	302	170	1.1147	1.1656	1.2941	7875.50
8	100	436	302	170	1.2294	1.3311	1.5882	7922.78
9	200	436	302	170	1.4587	1.6623	2.1765	8017.37
10	500	436	302	170	2.1468	2.6556	3.9412	8301.12

6. Conclusion

Using geometric programming method, we develop a total relevant annual inventory cost with uncertain lead time and varying holding cost in a fuzzy sense. Lead time is taken as a trapezoidal fuzzy number. Finally, the proposed model has been verified by the numerical example along with the sensitivity analysis. In the sensitivity analysis we obtain the optimal solution for varying β. From the above table, it implies that the dealers will decrease the value of β, and then only we get the minimum total annual relevant cost. In the future study, we introduce the decision variables and various types of constraints in this proposed model to obtain the optimal total cost.

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