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**Performance Investigation of Force Board Apparatus in the Study of Law of Parallelogram of Coplanar Concurrent Forces**

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**Abstract**

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Coplanar, Concurrent, Force

The basic purpose of this paper is to conclude that the Triangle law and Parallelogram law of forces are really different or the same. Author uses the same data to verify both these laws of forces and determine the percentage error in both the cases. At the same time, the same results are compared with the analytical one. Here, author assumes that the Force board apparatus is error free and every reading/data is taken with great care to minimize the human error upto a great extent. The paper is useful especially for B.Tech. / M.Tech./ and research scholar working in the field of machine design.

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**1. Introduction**

For a body is in equilibrium then the equilibrium conditions are that the resultant force (R) and resultant couple (M) are zero; i.e.  $\Sigma F = 0$  and  $\Sigma M = 0$ . These are two necessary and sufficient conditions of equilibrium. The couple of a force has a tendency to rotate an object about some point. The Physical Significance of a couple of force may be recognized as follows; A force acting on a rigid body has two effects: first it has a tendency to accelerate the object and to cause the object to rotate. A pure moment or Couple or torque is a rotational force has a tendency to induce rotation without translation. A pure moment is a vector quantity – it has magnitude and direction. When we solve problems with constraints, we are nearly always interested in analyzing forces in a structure having many parts, or the motion of a machine with a number of separate moving components. Solving such type of problem is very complicated because of the large number of forces involved and the number of equations that must be solved to find them. We draw free body diagrams of the bodies to get correct answers. Thomas [1] published his work related to vehicle raise and lower mechanisms. Razzaghi [2] develop electrical driven jack for raising and lowering of vehicle from ground level. Patil et. al. [3] suggested a screw jack to reduce the human effort. Tarachand [4] suggest the techniques to optimize the efficiency of screw jack by varying helix angle. Imani et al. [5] is worked with the problem of stochastic control of gene regulatory networks (GRNs) observed indirectly through noisy measurements. The performance of the resulting algorithm has been studied through a comprehensive set of numerical experiments. Imani M., et al.[6] worked on Partially-observed Boolean dynamical systems. Ghoreishi et. al. [7] gave a novel uncertainty propagation approach to multidisciplinary

system. McClenny, et al.[8] worked on that the Gene regulatory networks for governing the function of key cellular processes. Imani, and Braga-Neto [9] gave a framework for the measurement of Partially Observed Boolean Dynamical Systems. We have further consulted a number of references including [10-23] in the field of friction forces and lifting machine to complete the work.

## 2. Theory

A force board apparatus is shown in Figure-1. According to law of triangle law of force -if three coplanar forces acting at a point of a body and they can be represented in magnitude and direction by the three sides of the triangle taken in order then the force will be in equilibrium. In other words, If two forces acting on a point of a body can be represented in magnitude and direction by the two sides of the triangle taken in order then third side of the triangle will give their resultant in opposite direction. According to the law of Parallelogram of forces- if two forces acting at a point of a body can be drawn in magnitude and direction by the two sides of a parallelogram drawn from any point then the diagonal of the parallelogram drawn from the same point will give their resultant in its magnitude and direction.

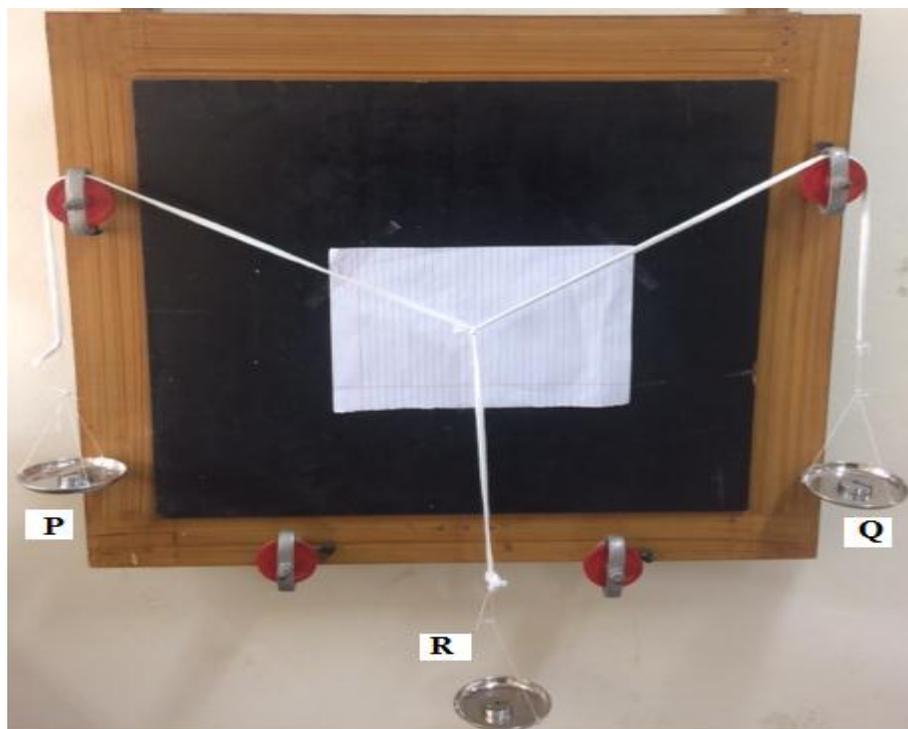


Figure-1: Force Board Apparatus

## 3. Methodology

Step-1: We fixed a paper sheet with the help of cello tape on the board such that the paper is parallel to the edge of the force board.

Step-2: We passed one thread over the pulleys carrying a pan at its each end. Now a second thread is tied its one end at the middle of the first one and it has a pan at its other end.

Step-3: Now we placed the equal weights in the pans in such a manner that the small knot comes approximately in the centre. There are equal angles between any two forces at the central knot. This has been done to minimize the human error in angle measuring.

Step-4: We marked lines of forces represented by thread without disturbing the equilibrium of the system and noted down the magnitude of forces i.e. self Pan Weight + Added additional Weight.

Step-5: Now, we remove the paper from the force board and produce the line to meet at point O. We used Bow's notation to name the force P, Q, R as AB, BC, and CA.

Ste-6: We took a suitable scale and draw the line  $ab$  parallel to force  $P$  and cut it equal to the magnitude of  $P$ . From  $b$  draw the line  $bc$  parallel to force  $Q$  and cut it equal to the magnitude of  $Q$  (Figure-2). Calculate the magnitude of  $ca$  i.e.,  $R_1$  which will be equal to the third force  $R$  which proves the triangle law of forces. If  $R_1$  magnitude is different from original magnitude of  $R$ , the percentage error is found as follows:  
Percentage error =  $(R - R_1/R) * 100$

### 3.1 Verification of Triangle Law of Forces

#### Graphical Method

According to Figure-2 (b), draw  $ab$  parallel to force  $P$  in suitable scale with the use of set square and then from  $b$  draw  $bc$  parallel to force  $Q$ . The closing side of triangle represents the force  $R_1$  which should be equal to force  $R$ . Note, the difference in  $R_1$  and  $R$  shows the graphical error.

#### Analytical Method

We Measure angles  $\alpha$ ,  $\beta$  and  $\gamma$  and by using Lami's theorem check the following relation

$$P/\sin \alpha = Q/\sin \beta = R_1/\sin \gamma$$

### 3.2 Verification of Parallelogram Law Of Forces

#### Graphical Method

According to Figure-2 (c), take  $OA = P$  and  $OB = Q$  in suitable scale. From  $A$  draw  $AC'$  parallel to  $OB$  and  $BC'$  parallel to  $OA$ .  $R_1$  represents the resultant of force  $P$  and  $Q$ . As the system is in equilibrium it must be equal to  $R$ . Noted down the magnitudes of  $R$  and  $R_1$  which are in opposite direction and the difference between  $R_1$  and  $R$  represents the graphical error.

#### Analytical Method

Measure angles  $\theta_1$  and by using resultant formula, calculate  $R_2$

$$R_2 = \sqrt{P^2 + Q^2 + 2PQ \cos \theta_1}$$

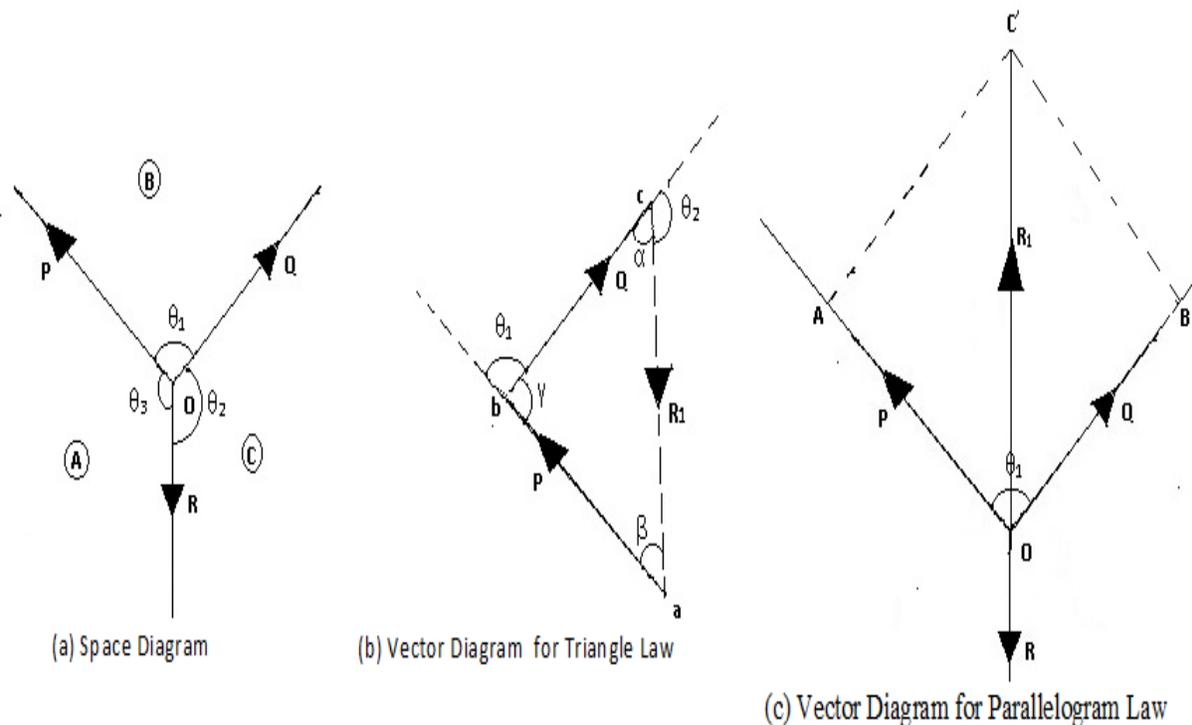


Figure-2: Verification of Triangle and Parallelogram Law of Forces

#### 4. Observations and Notations Used

P = load in Left string,

Q = load in Right string,

R = load in third string which is downward,

R1 = Resultant Force of 'P' and 'Q' by Experimental method using Triangle Law or Parallelogram Law of forces as the case may be,

R2 = Resultant Force by Analytical method using Triangle Law or Parallelogram Law of ,

$$= P (\sin\theta_1 / \sin\theta_2) = Q (\sin\theta_1 / \sin\theta_3) \text{ [using Lami's theorem],}$$

$$= \sqrt{P^2+Q^2+2 PQ \cosine \theta_1} \text{ [using Parallelogram Law of forces].}$$

%Error (E) = %age error by Experimental method =  $[(R-R1) / R] * 100$ ,

%Error (A) = %age error by Analytical method=  $[(R-R2) / R] * 100$ ,

Table-1: Observation Table to vafity the law of Parallelogram of coplanar concurrent forces

Law	S.N.	P (gm)	Q(gm)	R(gm)	$\theta_1$ (degrees)	$\theta_2$ (degrees)	$\theta_3$ (degrees)	R1(gm)	R2(gm)	%age Error(E)	Average Error
Triangle Law	1	50	50	50	120	120	120	50	50	0.0	2.97
	2	60	60	60	120	120	120	60	60	0.0	
	3	70	70	70	120	120	120	65	70	7.1	
	4	80	80	80	120	120	120	85	80	-6.3	
	5	90	90	90	120	120	120	95	90	-5.6	
	6	100	100	100	120	120	120	95	100	5.0	
	7	110	110	110	120	120	120	105	110	4.5	
	8	120	120	120	120	120	120	125	120	-4.2	
	9	150	150	150	120	120	120	150	150	0.0	
	10	200	200	200	120	120	120	200	200	0.0	
	11	250	250	250	120	120	120	250	250	0.0	
Parallelogram Law	1	50	50	50	120	120	120	55	50	-10.0	4.64
	2	60	60	60	120	120	120	65	60	-8.3	
	3	70	70	70	120	120	120	65	70	7.1	
	4	80	80	80	120	120	120	85	80	-6.3	
	5	90	90	90	120	120	120	95	90	-5.6	
	6	100	100	100	120	120	120	95	100	5.0	
	7	110	110	110	120	120	120	105	110	4.5	
	8	120	120	120	120	120	120	125	120	-4.2	
	9	150	150	150	120	120	120	150	150	0.0	
	10	200	200	200	120	120	120	200	200	0.0	
	11	250	250	250	120	120	120	250	250	0.0	

#### 5. Results and Discussion

As per the methodology discussed, the experiment was conducted in engineering mechanics laboratory and all the detailed results along with diagrams obtained experimentally are with the author. A brief summary of the results is listed in Table-1.the average percentage error in verifying the law of triangle is 2.97% while with the same data and same environment, it is 4.64% in verifying the parallelogram law of coplanar concurrent forces. In this way, both law of triangle and law of parallelogram of coplanar concurrent forces are proved.

#### 6. Conclusion

For the verification of triangle law and parallelogram law of coplanar concurrent forces, we took angle between any two forces equal to  $120^\circ$  to minimize the human error in angle measurement. We have repeated the same observations for verification of both the laws. In this way, we choose analytical error to be zero percent. We can observe from the results listed in Table-1 that there is less error in case of triangle law and error is increasing as the number of forces increased. This error will be increasing in case of polygon forces verification. Although, this error may be further reduced if we keep some important points in our mind like: there should not be any knot except central one in the string, Weight should be placed gently, While plotting the points on the sheets the thread should not

be disturbed, Threads and weights should not touch the board, Force Board should be vertical and while calculating the total force in each the weight of the pan should be added to the weight put into the pan.

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