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A Note on Analytic Mean Prime Labeling

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Abstract

Analytic mean prime labeling of a graph is the labeling of the vertices with {0,1,2---,p-1} and the edges with mean of the absolute difference of the squares of the labels of the incident vertices if square difference is even or mean of the absolute difference of the squares of the labels of the incident vertices and one if square difference is odd. The greatest common incidence number of a vertex (***gcin***) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the ***gcin*** of each vertex of degree greater than one is one, then the graph admits analytic mean prime labeling. Here we investigate tortoise graph, umbrella graph, fan graph, triangular belt, and some cycle related graphs for analytic mean prime labeling.

Keywords:

Graph Labeling;
Square Difference;
Greatest common incidence number;
Prime labeling;
Analytic mean.

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1. Introduction

All graphs in this paper are simple, finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q [2]. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges [3]. Some basic notations and definitions are taken from [2],[3] and [4] . Some basic concepts are taken from [1] and [2]. The analytic mean labeling was defined by T Tharmaraj and P Sarasija in [5]. In this paper we proved that tortoise graph, umbrella graph, fan graph, triangular belt, and some cycle related graphs admit analytic mean prime labeling.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

2. Research Method

Definition 2.1 Let G = (V, E) be a graph with p vertices and q edges. Define a bijection

f : V(G) → {0,1,2,3,---,p-1} by $f(v_i) = i-1$, $1 \le i \le p$. Define a 1-1 mapping f^*_{ampl} : E(G) → set of natural numbers N by

$f^*_{ampl}(uv) = \frac{|f(u)^2 - f(v)^2|}{2}$, when $f(u)^2 - f(v)^2$ is even [5].

$f^*_{ampl}(uv) = \frac{|f(u)^2 - f(v)^2|+1}{2}$, when $f(u)^2 - f(v)^2$ is odd [5].

The induced function f^*_{ampl} is said to be analytic mean prime labeling, if the ***gcin*** of each vertex of degree at least 2, is 1.

Definition 2.2 A graph which admits analytic mean prime labeling is called analytic mean prime graph.

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Definition 2.3 Let v_1,v_2---,v_n are the vertices of path P_n (n is even).Let G be the graph obtained by joining the vertices v_i and v_{n-i+1}($1 \leq i \leq \frac{n-2}{2}$) by edges. G is called tortoise graph.

Theorem 2.1 Tortoise graph admits analytic mean prime labeling, if n is even.

Proof: Let G be the graph and let v_1,v_2,---,v_n are the vertices of G

Here |V(G)| = n and |E(G)| = $\frac{3n-4}{2}$.

Define a function f : V → {0,1,2,3,---,n-1 } by

$f(v_i) = i-1$, i = 1,2,---,n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ampl} is defined as follows

$f^*_{ampl}\,(v_i\ v_{i+1})$	= i,	i = 1,2,---,n-1
$f^*_{ampl}\,(v_i\ v_{n-i+1})$	= $\frac{n^2-2ni+2i}{2}$,	i = 1,2,---,$\frac{n-2}{2}$

Clearly f^*_{ampl} is an injection.

gcin of (v_1)	= gcd of $\{f^*_{ampl}\,(v_1\ v_2)\,,\ f^*_{ampl}\,(v_1\ v_n)\}$	
	= gcd of { 1, $\frac{n^2-2n+2}{2}$}= 1.	
gcin of (v_{i+1})	= gcd of $\{f^*_{ampl}\,(v_i\ v_{i+1})\,,\ f^*_{ampl}\,(v_{i+1}\ v_{i+2})\}$	
	= gcd of { i, i+1}	
	= 1,	i = 1,2,---,n-2
gcin of (v_n)	= gcd of $\{f^*_{ampl}\,(v_1\ v_n)\,,\ f^*_{ampl}\,(v_{n-1}\ v_n)\}$	
	= gcd of {$\frac{n^2-2n+2}{2}$, n-1}= 1.	

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G , admits analytic mean prime labeling. ■

Definition 2.4 Let α be a sequence of n symbols of S. We will construct a graph by tiling n square blocks side by side, with their position indicated by α. We will denote the resulting graph by TB(α) and is called triangular belt.

Theorem 2.2 Triangular belt admits analytic mean prime labeling.

Proof: Let G = TB(α), where α = (↑↑---↑) be the graph and let v_1,v_2,---,v_{2n} are the vertices of G

Here |V(G)| = 2n and |E(G)| = 4n-3.

Define a function f : V → {0,1,2,3,---,2n-1 } by

$f(v_i) = i-1$, i = 1,2,---,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ampl} is defined as follows

$f^*_{ampl}\,(v_i\ v_{i+1})$	= i,	i = 1,2,---,2n-1
$f^*_{ampl}\,(v_i\ v_{2n-i+1})$	= $2n^2$-2ni+i,	i = 1,2,---,n-1
$f^*_{ampl}\,(v_{i+1}\ v_{2n-i+1})$	= $2n^2$-2ni,	i = 1,2,---,n-1

Clearly f^*_{ampl} is an injection.

gcin of (v_1)	= gcd of $\{f^*_{ampl}\,(v_1\ v_2)\,,\ f^*_{ampl}\,(v_1\ v_{2n})\}$	
	= gcd of { 1, $2n^2$-2n+1}= 1.	
gcin of (v_{i+1})	= 1,	i = 1,2,---,2n-2
gcin of (v_{2n})	= gcd of $\{f^*_{ampl}\,(v_1\ v_{2n})\,,\ f^*_{ampl}\,(v_{2n-1}\ v_{2n})\}$	
	= gcd of { $2n^2$-2n+1, 2n-1}	
	= gcd of {n, 2n-1}= 1.	

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G , admits analytic mean prime labeling. ■

Theorem 2.3 Fan graph[4] admits analytic mean prime labeling, if n is odd.

Proof: Let G = F_n and let v_1,v_2,----------------,v_{n+1} are the vertices of G

Here |V(G)| = n+1 and |E(G)| =2n-1.

Define a function f : V → {0,1,2,3,---,n } by

$f(v_i) = i-1$, i = 1,2,---,n+1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ampl} is defined as follows

$f^*_{ampl}\,(v_i\ v_{i+1})$	= i,	i = 1,2,---,n-1
$f^*_{ampl}\,(v_{n+1}\ v_{2i-1})$	= $\frac{n^2-(2i-2)^2+1}{2}$,	i = 1,2,---,$\frac{n+1}{2}$
$f^*_{ampl}\,(v_{n+1}\ v_{2i})$	= $\frac{n^2-(2i-1)^2}{2}$,	i = 1,2,---,$\frac{n-1}{2}$

Clearly f^*_{ampl} is an injection.

gcin of (v_1) = gcd of $\{f^*_{ampl}\ (v_1\ v_2)\ ,\ f^*_{ampl}\ (v_1\ v_{n+1})\ \}$

= gcd of $\{\ 1, \frac{n^2+1}{2}\}$= 1.

gcin of (v_{i+1}) = 1, i = 1,2,---,n-1.

gcin of (v_{n+1}) = gcd of $\{f^*_{ampl}\ (v_1\ v_{n+1})\ ,\ f^*_{ampl}\ (v_2\ v_{n+1})\ \}$[1]

= gcd of $\{\frac{n^2+1}{2}, \frac{n^2-1}{2}, \}$= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G , admits analytic mean prime labeling. ■

Theorem 2.4 Let G be the graph obtained by duplicating an edge in cycle C_n. G admits analytic mean prime labeling, if n is even and (n-2) $\not\equiv$ 0(mod6).

Proof: Let G be the graph and let v_1,v_2,---,v_{n+2} are the vertices of G

Here |V(G)| = n+2 and |E(G)| = n+3.

Define a function f : V → {0,1,2,3,---,n+1 } by

$f(v_i)$ = i-1 , i = 1,2,---,n+2

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ampl} is defined as follows

$f^*_{ampl}\ (v_i\ v_{i+1})$ = i, i = 1,2,---,n+1

$f^*_{ampl}\ (v_1\ v_n)$ = $\frac{n^2-2n+2}{2}$.

$f^*_{ampl}\ (v_{n+2}\ v_3)$ = $\frac{n^2+2n-2}{2}$.

Clearly f^*_{ampl} is an injection.

gcin of (v_1) = gcd of $\{f^*_{ampl}\ (v_1\ v_2)\ ,\ f^*_{ampl}\ (v_1\ v_n)\ \}$

= 1

gcin of (v_{i+1}) = 1, i = 1,2,---,n.

gcin of (v_{n+2}) = gcd of $\{f^*_{ampl}\ (v_{n+1}\ v_{n+2})\ ,\ f^*_{ampl}\ (v_{n+2}\ v_3)\ \}$

= gcd of $\{\ n+1, \frac{n^2+2n-2}{2}\ \}$

= 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G , admits analytic mean prime labeling. ■

Theorem 2.5 Let G be the graph obtained by duplicating an edge in cycle C_n by a vertex. G admits analytic mean prime labeling, if n is odd.

Proof: Let G be the graph and let v_1,v_2,---,v_{n+1} are the vertices of G

Here |V(G)| = n+1 and |E(G)| = n+2.

Define a function f : V → {0,1,2,3,---,n } by

$f(v_i)$ = i-1 , i = 1,2,---,n+1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ampl} is defined as follows

$f^*_{ampl}\ (v_i\ v_{i+1})$ = i, i = 1,2,---,n.

$f^*_{ampl}\ (v_1\ v_n)$ = $\frac{n^2-2n+1}{2}$.

$f^*_{ampl}\ (v_{n+1}\ v_1)$ = $\frac{n^2+1}{2}$.

Clearly f^*_{ampl} is an injection.

gcin of (v_1) = gcd of $\{f^*_{ampl}\ (v_1\ v_2)\ ,\ f^*_{ampl}\ (v_1\ v_n)\ \}$

= 1

gcin of (v_{i+1}) = 1, i = 1,2,---,n-1.

gcin of (v_{n+2}) = gcd of $\{f^*_{ampl}\ (v_{n+1}\ v_n)\ ,\ f^*_{ampl}\ (v_{n+1}\ v_1)\ \}$

= gcd of $\{\ n, \frac{n^2+1}{2}\ \}$

= 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G , admits analytic mean prime labeling. ■

Theorem 2.6 Let G be the graph obtained by switching a vertex in cycle C_n. G admits analytic mean prime labeling, if n is even and n > 4.

Proof: Let G be the graph and let v_1,v_2,---,v_n are the vertices of G

Here |V(G)| = n and |E(G)| = 2n-5.

Define a function f : V → {0,1,2,3,---,n-1 } by

$f(v_i)$ = i-1 , i = 1,2,---,n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ampl} is defined as follows

$f^*_{ampl}(v_i\, v_{i+1})$ = i, i = 1,2,---,n-2.

$f^*_{ampl}(v_{2i}\, v_n)$ $= \frac{n^2-2n-4i^2+4i}{2}$. i = 1,2,---,$\frac{n-2}{2}$

$f^*_{ampl}(v_{2i+1}\, v_n)$ $= \frac{n^2-2n-4i^2+2}{2}$. i = 1,2,---,$\frac{n-4}{2}$

Clearly f^*_{ampl} is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,---,n-3.

gcin of (v_n) = gcd of $\{f^*_{ampl}(v_2\, v_n), f^*_{ampl}(v_n\, v_3)\}$

= gcd of $\{ \frac{n^2-2n}{2}, \frac{n^2-2n-2}{2} \}$

= 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G , admits analytic mean prime labeling. ■

Theorem 2.7 Umbrella graph U(m,n) admits analytic mean prime labeling, if m is odd and n ≤ m-2.

Proof: Let G = U(m,n) and let v_1, v_2,---,v_{m+n} are the vertices of G

Here |V(G)| = m+n and |E(G)| =2m+n-2.

Define a function f : V → {0,1,2,3,---,m+n-1 } by

$f(v_i)$ = i-1 , i = 1,2,---,m+n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ampl} is defined as follows

$f^*_{ampl}(v_i\, v_{i+1})$ = i, i = 1,2,---,m+n-1

$f^*_{ampl}(v_{m+1}\, v_{2i-1})$ $= \frac{m^2-(2i-2)^2+1}{2}$, i = 1,2,---,$\frac{m+1}{2}$

$f^*_{ampl}(v_{m+1}\, v_{2i})$ $= \frac{m^2-(2i-1)^2}{2}$, i = 1,2,---,$\frac{m-1}{2}$

Clearly f^*_{ampl} is an injection.

gcin of (v_1) = gcd of $\{f^*_{ampl}(v_1\, v_2), f^*_{ampl}(v_1\, v_{m+1})\}$

= gcd of $\{ 1, \frac{m^2+1}{2}\}$= 1.

gcin of (v_{i+1}) = 1, i = 1,2,---,m+n-2.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G , admits analytic mean prime labeling. ■

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