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Unsteady free convection flow of a visco-elastic fluid past an impulsively started porous wall with heat transfer for the lower value of Prandtl number

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Abstract:

Heat transfer in the unsteady free convection flow of a visco-elastic fluid past an impulsively started porous wall for a lower prandtl number has been studied. The constitutive equations of the problem have been formulated and solved by perturbation technique applying the boundary conditions. Expressions for velocity, temperature, skin friction and rate of heat transfer have been obtained. Graphs are plotted to present the velocity and temperature profiles. It is observed that the increase in the Prandtl number decreases the mean as well as the transient temperature of the fluid.

Keywords: Heat transfer, porous medium, perturbation technique.

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Introduction:

The unsteady flow problems are very important in various technological fields like aeronautics, atomic power, chemical engineering and space science, etc. Pohlausen[1] developed an analytical solution and Ostrach[2] a numerical solution for the isothermal vertical plate at steady state. Gebhart[3] developed an approximate solution for the transient behavior with a constant heat flux density at the plate.
Soundalgekar and Pop[4] have studied the unsteady flow past an infinite vertical porous plate with constant or variable suction. Agarwal and Upmanyu[5] have analysed the heat transfer in the presence of temperature dependent heat sources in a second order fluid flowing over a flat plate with uniform suction.

Menold and Yang[6] have analysed the unsteady free convection flow past an infinite plate where as Nanda Sharma[7] investigated the unsteady free convection flow past a semi-infinite vertical plate. Physically the heat generating source or heat absorbing sink is considered to be due to combustion process or after shut-down cooling problem associated with nuclear reaction. In case of chemical reactions, exothermic and endothermic cases may simulated sources or sinks respectively within the flow.

Singh and Naveen Kumar[8] have studied the free convection flow of an incompressible viscous fluid past an exponentially accelerated infinite vertical plate. Yadav and Singh[9] have studied the impulsive motion of a porous flat plate in an elastico-viscous liquid under the influence of uniform transverse magnetic field. Teipel[10] has studied the problem of the impulsive motion of a flat plate in a visco-elastic field. Free convection effect on the flow of an elastico-viscous fluid past an exponentially accelerated vertical plate has been studied by Das and Biswal[11].

Das and Ojha[12] have studied the MHD unsteady free convection effect on the flow past an exponentially accelerated vertical plate. The unsteady free convection flow and heat transfer of avisco-elastic fluid past an impulsively started porous flat plate with heat and source/sink have analysed by Biswal Mahalik[13]. Datta, Biswal and Sahoo[14] have been studied magneto-hydrodynamic unsteady free convection flow and heat transfer of a visco-elastic fluid past an impulsively started flat plate with heat sources/sinks. Biswal[15] has analysed the unsteady free convection flow and heat transfer of a visco-elastic fluid past an impulsively started porous wall.

Our aim here is to study the unsteady free convection flow of a visco-elastic fluid past an impulsively started porous wall with heat transfer at the lower Prandtl number.

Formulation of the problems:

The second order approximation of the general constitutive equations given by Rivlin-Ericksen can be written as,

$T = - PI + \mu A_1 + \alpha A_1^2 + \beta A_2$ (1)

Where T is the stress tensor

P is the pressure

I is the unit tensor

A_1 and A_2 are the first two Rivlin-Ericksen tensors.

μ, α and β are three material constants.

A_1 and A_2 are given by the symmetric matrices defined by

$$A_1 = \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \quad (2)$$

$$A_2 = \frac{\partial}{\partial x_j}\left(\frac{Dv_i}{Dt}\right) + \frac{\partial}{\partial x_j}\left(\frac{Dv_j}{Dt}\right) + 2\frac{\partial v_m}{\partial x_i}.\frac{\partial v_m}{\partial x_j} \quad (3)$$

Where, (i. j, m = 1, 2, 3) and v_i and v_j are the components of velocity.

Here the wall is porous and horizontal. We take x' axis along the flat wall and y' axis normal to it. u' is the velocity of the fluid along x' axis and v' is the velocity along y' axis. Consequently u' is a function of y' and t' but v' is independent of y'.

Let a constant impulsive velocity u be given to the plate in its own plane. For the boundary condition it is assumed that there is no slip at the wall. Thus the flow is governed by the following equations.

Equation of Continuity :

$$\frac{\partial V'}{\partial Y'} = 0 \Rightarrow V' = \text{constant} = -V_0 \quad (4)$$

We take V_0 as the suction velocity and negative sign indicates that suction is towards the plate.

Equation of Motion :

$$\frac{\partial u'}{\partial t'} + V'\frac{\partial u'}{\partial y'} = v\frac{\partial^2 u'}{\partial y'^2} + \frac{K_0}{\rho}\frac{\partial^3 u'}{\partial t'\partial y^2} - \frac{v}{K'}u' + g\beta\left(T' - T'_\infty\right) \quad (5)$$

Equation of Energy :

$$V'\frac{\partial T'}{\partial y'} + \frac{\partial T'}{\partial t'} = \frac{K}{\rho C_P}\frac{\partial^2 T'}{\partial y'^2} + S'\left(T' - T'_\infty\right) \quad (6)$$

The boundary conditions imposed are

$$\left.\begin{array}{l} t' \le 0,\ u' = 0,\ T' = T'_\infty,\ \text{for } y' \ge 0 \\ t' \le 0,\ u' = U,\ T' = T'_\infty(T'_w - T'_\infty)e^{i\omega t'}\ \text{for } y' = 0 \\ u' \to 0,\ T' \to T'_\infty,\quad \text{for } y' \to \infty \end{array}\right\} \quad (7)$$

We introduce the following non-dimensional quantities

$$\left.\begin{array}{l} y = \dfrac{y'}{\sqrt{vT}}, u = \dfrac{u'}{U}, t = \dfrac{t'}{T}, R_c = \dfrac{K_0}{\eta_0 T} \\ V = \dfrac{v'}{U}, \theta = \dfrac{T' - T'_\infty}{T'_w - T'_\infty},\ P_r = \dfrac{\nu\rho C_p}{K} \\ G_r = \dfrac{vg\beta(T'_w - T'_\omega)}{U^3},\ S = \dfrac{S'v_1}{U^2} \\ \text{and}\quad K^* = \dfrac{K^1 U^2}{v^2}, \end{array}\right\} \quad (8)$$

Consequently the equation of continuity, motion and energy into their corresponding non-dimensional form as,

$$\frac{\partial V}{\partial y} = 0 \quad (9)$$

$$\frac{\partial u}{\partial t}+V\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+R_c\frac{\partial^3 u}{\partial t\partial y^2}-\frac{1}{K^*}u+Gr\theta \quad (10)$$

$$\frac{\partial \theta}{\partial t}+V\frac{\partial \theta}{\partial y}=\frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2}+S\theta \quad (11)$$

The dimensionless form of the boundary conditions are given by,

$$T>0\begin{cases} u=1,\ \theta=1+\varepsilon e^{i\omega t} & \text{at } y=0\\ u=0,\ \theta=0 & \text{at } y\to\infty \end{cases} \quad (12)$$

Solutions of the equations:

Solving equation-(9),we obtained

V= constant

For constant suction, let us take

$V=-V_0$ (13)

Where, the negative sign indicates that the suction is towards the plate.

By putting equation(13) in equation(10) and (11), we obtain

$$\frac{\partial u}{\partial t}-V_0\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+R_c\frac{\partial^3 u}{\partial t\partial y^2}-\frac{1}{K^*}u+G_r\theta \quad (14)$$

$$\frac{\partial \theta}{\partial t}-V_0\frac{\partial \theta}{\partial y}=\frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2}+S\theta \quad (15)$$

Equation(14) is a third order differential equation because of the presence of the elastic parameter Rc. It requires three boundary conditions to be solved, while the present problem provides only two. Therefore, we apply small parameter perturbation technique forwarded by Beard and Walters[16] and assumed as,

$u=u_0+\varepsilon e^{i\omega t}u_1$ (16)

$\theta=\theta_0+\varepsilon e^{i\omega t}\theta_1$ (17)

Now substituting(16)and(17) in equation(14) and (15) respectively and separating harmonic non- harmonic terms and neglecting the terms containing R_c^2,we obtain the zeroth and first order equations as,

Zeroth order equations:

$$u_0''+V_0u_0'-\frac{1}{K^*}u_0=-G_r\theta_0 \quad (18)$$

$$\frac{1}{P_r}\theta_0''+V_0\theta_0'+S\theta_0=0, \quad (19)$$

First order equations:

$$(1+i\omega R_c)u_1''+V_0u_1'-\left(i\omega+\frac{1}{K^*}\right)u_1=-G_r\theta_1 \quad (20)$$

$$\frac{1}{P_r}\theta_1''+V_0\theta_1'-(i\omega-S)\theta_1=0, \quad (21)$$

Where the Prime (’) denotes differentiation with respect to Y.

The boundary conditions given in equation (2.11) are further modified as

$$t>0\begin{cases} u_0=1,\ u_1=0,\ \theta_0=1,\ \theta_1=0 & \text{at } y=0\\ u_0=0,\ u_1=0,\ \theta_0=0,\ \theta_1=0 & \text{at } y\to\infty \end{cases} \quad (22)$$

Equation (18) - (21) are solved separately with the help of boundary conditions given in (22) to give.

$$\theta_0=e^{-a_1y} \quad (23)$$

$$\theta_1=e^{-a_3y} \quad (24)$$

$$u_0=\left[1+\frac{G_r}{a_1^2-a_1V_0-1/K^*}\right]e^{-a_5Y}-\left[\frac{G_r}{a_1^2-a_1V_0-1/K^*}\right]e^{-a_1Y} \quad (25)$$

and

$$u_1 = \left[\frac{G_r e^{-a_7 y}}{(1+i\omega R_c)a_3^2 - V_0 a_3 - \left(i\omega + \frac{1}{K^*}\right)} \right] - \left[\frac{G_r e^{-a_3 y}}{(1+i\omega R_c)a_3^2 - V_0 a_3 - \left(i\omega + \frac{1}{K^*}\right)} \right] \tag{26}$$

From equation (23) and (24), we obtain the expression for temperature as,

$$\theta = e^{-a_1 y} + \varepsilon e^{i\omega t} e^{-a_3 y} \tag{27}$$

Similarly, from equation(25) and (26),

$$u = \left[1 + \frac{G_r}{a_1^2 - a_1 V_0 - \frac{1}{K^*}} \right] e^{-a_5 Y} - \left[\frac{G_r}{a_1^2 - a_1 V_0 - \frac{1}{K^*}} \right] e^{-a_1 Y}$$

$$+\varepsilon e^{i\omega t} \left[\frac{G_r e^{-a_7 y}}{(1+i\omega R_c)a_3^2 - V_0 a_3 - \left(i\omega + \frac{1}{K^*}\right)} \right] - \left[\frac{G_r e^{-a_3 y}}{(1+i\omega R_c)a_3^2 - V_0 a_3 - \left(i\omega + \frac{1}{K^*}\right)} \right] \tag{28}$$

Separating harmonic and non-harmonic terms in equation (28) and (27), we obtain the real part of velocity and temperature as,

$$u(y, t) = u_0(y) + \varepsilon (M_r \, Cos \, \omega t - M_i \, Sin \, \omega t) \tag{29}$$

$$\theta(y, t) = \theta_0(y) + \varepsilon (\theta_r \, Cos \, \omega t - \theta_i \, Sin \, \omega t) \tag{30}$$

Where ,

$M_r = P_3 P_6 - Q_3 Q_6 - P_3 P_7 + Q_3 Q_7$

$M_i = P_3 Q_7 + Q_3 Q_7 - Q_3 P_6 - P_3 Q_6$

$$u_0 = P_5 e^{-a_5 Y} - P_4 e^{-a_1 y} \tag{31}$$

$$u_1 = M_r + i M_i \tag{32}$$

$\theta_r = P_7$

$\theta_I = Q_7$

$$\theta_0 = e^{-a_1 y} \tag{33}$$

$$\theta_1 = \theta_r + i\theta_I \tag{34}$$

Where the transient velocity and temperature for $\omega t = \frac{\pi}{2}$ are represented by,

$$u = u_0 - \varepsilon M_i \tag{35}$$

$$\theta = \theta_0 - \varepsilon \theta_i \tag{36}$$

Mean Skin friction:

The mean skin friction is given by,

$$\tau_m = \left.\frac{du_0}{dy}\right|_{y=0} + R_c \left.\frac{d^2 u_0}{dy^2}\right|_{y=0} \tag{37}$$

Skin friction:

The skin friction at the plate is given by,

$$\tau = \left.\frac{du}{dy}\right|_{y=0} + R_c \left.\frac{d^2 u}{dy^2}\right|_{y=0} \tag{38}$$

Mean rate of heat transfer:

The mean rate of heat transfer is given by the Nusselt number,

$$Nu_0 = -\left.\frac{\partial \theta_0}{\partial y}\right|_{y=0} \tag{39}$$

Rate of heat transfer:

The rate of heat transfer is given by the Nusselt number,

$$Nu = -\left.\frac{\partial\theta}{\partial y}\right|_{y=0} \tag{40}$$

Results and Discussion:

Numerical results for velocity, temperature, skin friction and Nusselt number are obtained.

The lowervalue of Prandtl number are chosen 0.1, 0.5 and 0.9.

Velocity profiles of the fluid motion through a porous flat plate are shown in the fig-1 and fig-2 .

Figure-1 exibits the effects of G_r and K^* on the mean velocity u_0 first rises and then falls as y increases for different values of G_r and K^*. With the increase of G_r ,u_0 increases at every point but the increase of K^* reduces u_0 of flow.

Figure-2 exibits the effects of G_r ,K^* and R_c on the transient velocity u.It is noticed that the transient velocity first rises and then falls as y increases for different values of G_r , K^* and R_c . With the increase in the value of G_r , u of the fluid increases but the increase in K^* decreases u of the fluid flow.

Temperature profiles of the fluid motion through a porous flat plate are shown in the fig-3 and fig-4 .

Figure-3 shows the effects of Prandtl number (P_r) on the temperature (θ_0) of the fluid .It is observed that θ_0 decreases with the increases of Prandtl number P_r .

Figure-4 shows the influence of P_r on the transient temperature θ of the fluid, while other parameters are kept constant . It is noticed that θ decreases with the rises of P_r .

Thus the nature of the mean temperature field and transient temperature field are the same .

Conclusion:

The study of above problem gives some conclusions. The increase of permeability factor K^* reduces both mean and transient velocity of visco-elastic fluid flow for a lower value of Prandtl number P_r. Similarly the increase in prandtl number P_r decreases the mean and transient temperature of the flow of fluid.

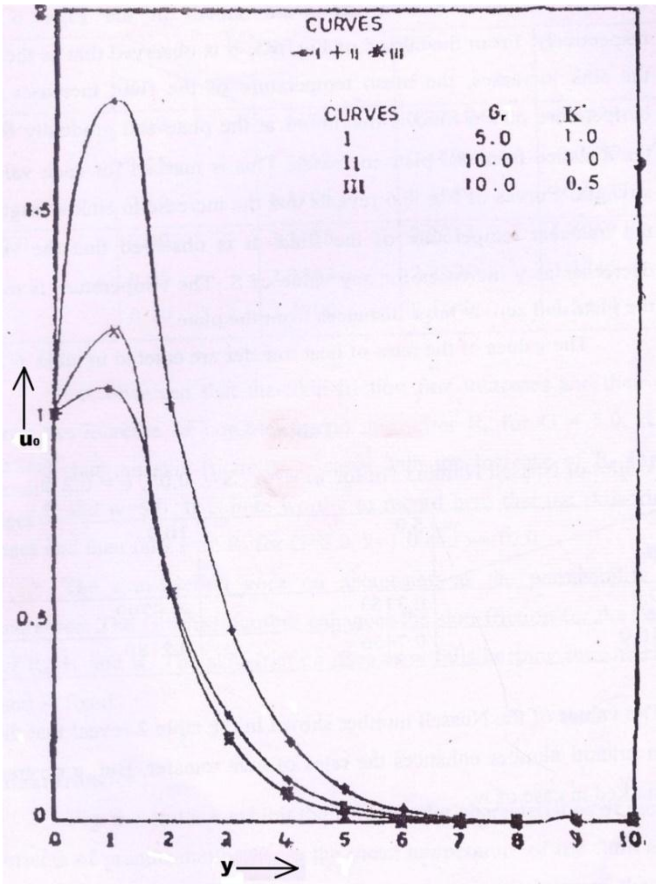

Fig-1:- Effect of G_r and K^* on mean velocity

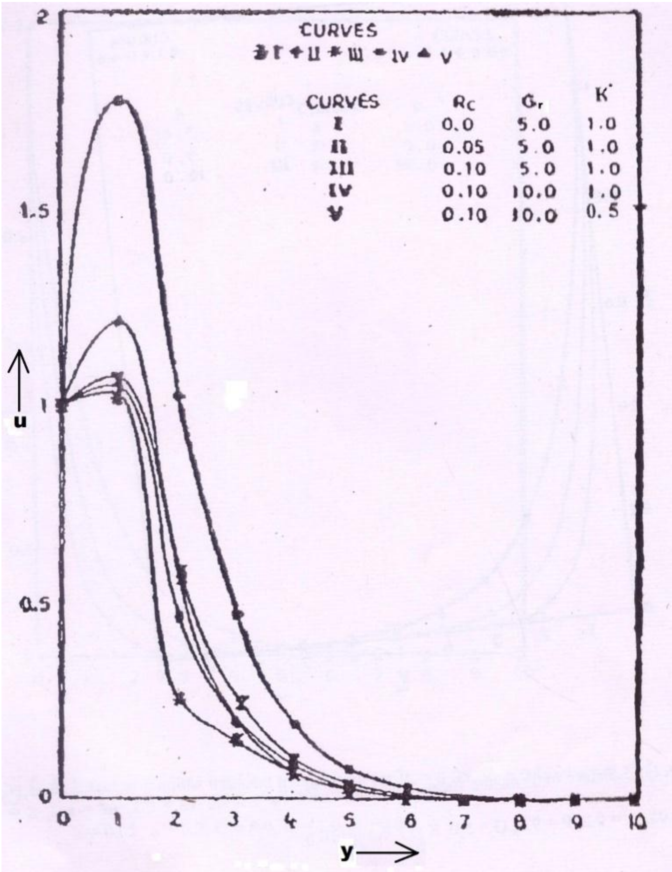

Fig-2:-Effect of Rc, G_r and K^* on transient velocity

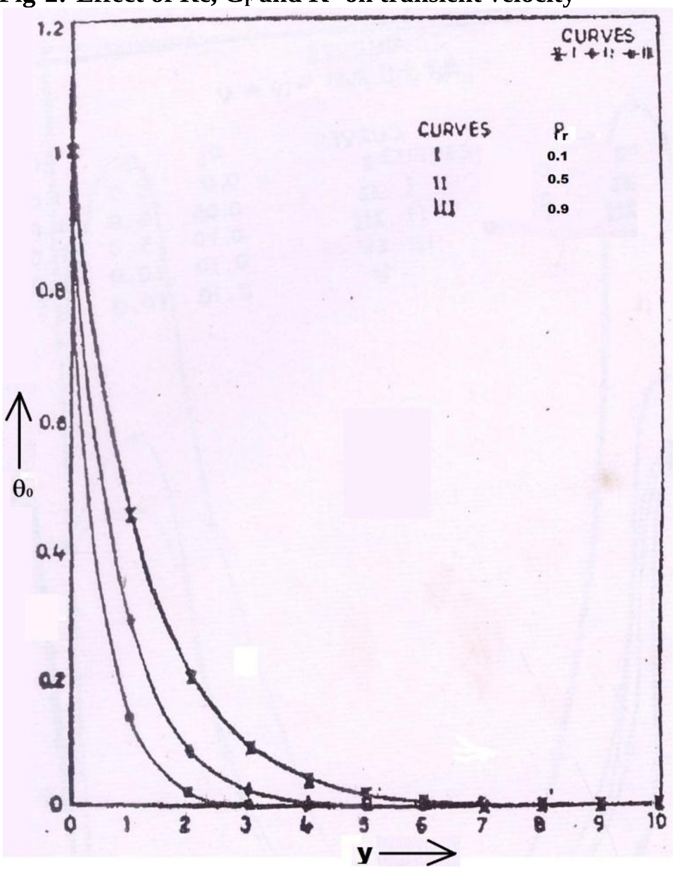

Fig-3:-Effect of P on mean temperature

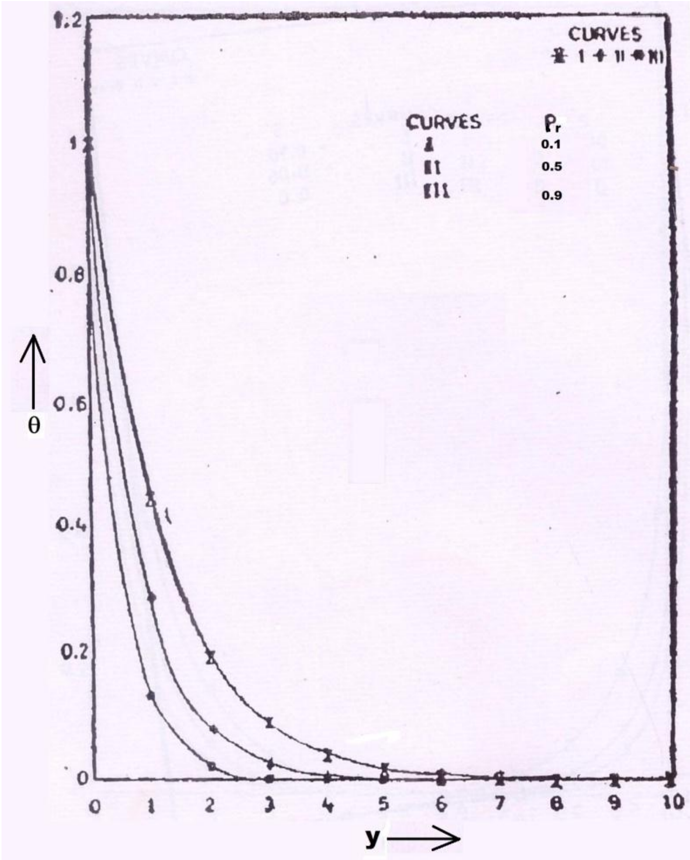

Fig-4:-Effect of P_r on transient temperature

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