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# STATISTICAL ANALYSIS OF STUDIES ON HEALTHCARE SECTORS USING QUEUING THEORY

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**Abstract:**

Queuing theory is a useful method to tackle congestion problems of majority of sectors. This approach is used to maintain the quality standards of service sectors also. The most recent trend of using queuing theory is in healthcare sectors and year by year, it is increasing drastically. The main motive of this research paper is to give overview of researches done on healthcare sectors using queuing theory approach. The literature surveypresented in this paper shows that the trend of using queuing theory in hospitals was rare before the year 2000 but this vogue has been increased recently. This leads to number of studies that have been done after the year 2000.Apart from that, this research paper also includes statistical analysis of distribution of focus of these researches on different departments of hospitals and it is evident that emergency department has remained a most used department independently among studies carried out in hospitals.

**Keywords:** Queuing theory, Quality in healthcare, Hospitals, Waiting time, Customers satisfaction.

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## 1. Introduction

The queueing theory was developed by A.K. Erlang in 1903 when he decided to implement this theory to eradicate the congestion problem of telephone traffic. Due to enormous number of calls, telephone operators were not able to handle this congestion and this need of proper management of queue problems kindled the need of queueing theory. Erlang made his first attempt to find the reasons of delay for a single operator at first. The result of this experiment was extended to find the delay for several operators and then it was successfully implemented on other general problems involving queues. Later different authors defined queuing theory with their own perception. Iverson (1993) categorized this theory in two parts. Bruin et al. (2007) considered queuing theory as a tool to increase operational efficiency. While Li et al. (2009) found queuing theory as an integral part of other management tool like Goal Programming. Mike and Wilson (2001) defined it as a mathematical approach to handle operations of management where waiting lines are present. Mehandiratta et al. (2011) discussed various application of queuing theory and defined it as a tool to minimize the service cost when there are several number of queues formations.

Queues are present everywhere in industries, hospitals, schools, book stores, banks, petrol pumps etc. Queue problems mainly arise because of two factors:

(i) Due to excess demand of the facility which leads to overcrowding of customers.

(ii) Due to very less demand of the facility which leads to idleness of a server.

If either case is detected in a system then providing sufficient number of servers or scheduling arrivals or both will help to make a system work in more efficient way.

## 2. Application of queueing theory

Queuing theory has multiple applications in every sector and examples are: hospitals, restaurants, department stores etc., queuing theory is used to reach adequate customer satisfaction level by serving thecustomers quickly and keeping cost to the minimum level. While in manufacturing units, it has an application to make the flow of work smooth by scheduling work in advance. Even at harbours, it has significant role to determine the proper numbers of docks necessary for trucks and ships to work without waiting more than a specific amount of time. Queuing theory has been used extensively in industries to evaluate required number of repairmen for taking care of machines that stop working at random times. Besides this it is used to resolve the right wage

incentive plans for a worker based on his existent working capability. It also has its application in finding solutions for inventory control problems and scheduling fleets for transport of raw materials from one place to another. Even scheduling jobs in production plant having number of channels comes under the influence of queuing theory.

## 3. General structure of a queueing theory

(i) Arrival Distribution: The pattern in which customers come at service facility to get served is called arrival distribution. Mostly, customers arrive in a random manner but a specific time interval is set to estimate their coming behaviour. The most common behaviour of customers arriving at facility centre is demonstrated by Poisson distribution by taking some assumption under consideration.

(ii) Service Distribution: The pattern in which customers leave facility centre after getting served is called service distribution. When service time is random then it can be best described by Exponential distribution. Service distribution helps to calibrate the parameter called service rate, which helps to calculate an average time to serve a single customer.

(iii) Service Channel: Based on requirement and nature of a facility, there are different types of service channel scenarios available. It can be a single service channel if the service is server oriented, for example doctor's clinic. Or it can be multi channels service as present in barber shop. Also, it can be series channel service if a customer passes through number of severs successively to make a final product.

(iv) Service Discipline: It is a rule by which customers are selected for service. 'First Come First Serve' discipline is most common in this category in which those customers are served with priority who join the queue first. Other less common disciplines are 'Last Come First Serve', 'Service In Random Order' etc.

(v) Capacity of the system: It is the maximum number of customers those a system can hold for a moment. Either a system can be of infinite capacity or finite capacity. If the system is of finite capacity then it can only accommodate limited number of customers and remaining customers can only enter the system if existing customer leaves the system.

(vi) Population: It serves as a source for arrival pattern. It takes into account potential customers who can enter the system to avail its service. If population is considered to be finite then only specific number of customers would like to use service. But if the calling source is

infinite then that means arrival of a customer will not affect the behaviour of upcoming customers.

(vii) Customers' Behaviour: Customer behaviour also determines the efficient working conditions of a queue system. If customer does not enter the queue and leaves the system then it is said to be balked and if he enters but decides to leave after some time then he is said to be have reneged. When two or more parallel line are in asystem and customer shifts lines in hope for getting served quickly then he is jockeying.

Figure 1 shows the general flow chart of Queuing system.

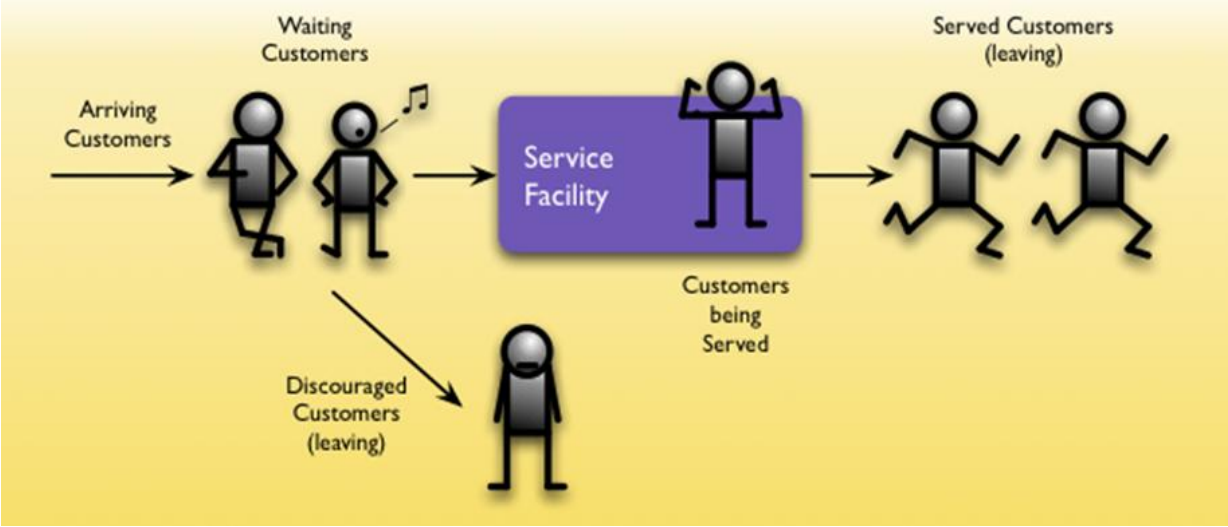


**Figure 1 General flow chart of Queuing system**

## 4. Queuing theory in Healthcare sectors

The operational research has been helping healthcare sectors to deal with their problems related to management in a better way. Healthcare sectors play a vital role to hold up the economy of the country. Earlier, people were more bothered about getting treatment from hospitals and it was rare when someone had actually raised a finger on a quality of the service provided by hospitals. But in recent years, the pace of expensive services has been introduced to healthcare units. With the introduction of these high-end facilities, people are more concerned about whether their money has been spent on worthy things or not. Now people take into account every second they

spend to avail a service. This attitude has initiated a movement which is forcing healthcare sectors to maintain their quality standards.

The most common problem faced by hospitals is overcrowding of patients. This congestion is the main measure of the quality of service that is provided by the hospitals. People don't wait to get served when they find a long queue is already present to avail that service. To rectify this problem, queuing theory has been successfully used to eradicate the problem of congestion. Queuing theory is used to minimise the cost of waiting and helps to run the system without formation of long queues.

Figure 2 shows the flow pattern of patients in hospitals.

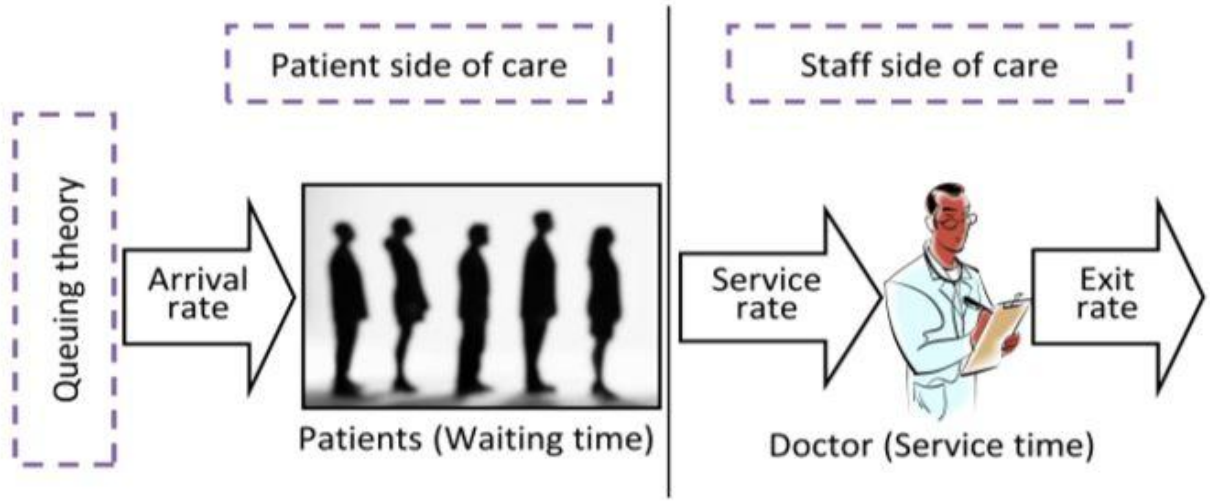


**Figure 2 General flow chart of Queue system in hospitals**

Total thirty numbers of research papers have been studied and summarised in this review paper to show the applications of queuing theory in healthcare sector. These studies have been done on various departments of hospitals for their efficient working and to find the bottlenecks of existing systems. These studies have been classified in to threesub-sections.

1. Emergency & Ambulance services
2. Intensive Care Unit ( ICU) & Out-Patient Department (OPD)
3. Other Departments

### 4.1. Emergency & Ambulanceservices

Emergency Department (ED) is also known as a casualty department and it is a medical facility to facilitate patients coming without a prior appointment. Patients coming to ED are generally taken in by ambulance. But some patients don't take the help of ambulance service provided by hospital and come by themselves. Ambulance service is provided by hospital to help patients to reach hospital in time. Most first-aid treatments are given to patients inside the ambulance while traveling towards a hospital. Patients enrolled by ambulance are admitted in ED for initial treatment in an attempt to save patients' life.

Thompson et al. (1996) conducted a study in order to find the actual and perceived waiting time taking customer satisfaction under consideration. Telephone calls were made to random sample of patients who were admitted in emergency department of the hospital for last two to four weeks. These patients were asked question about the overall quality of the service they were given to, including waiting time, expressive quality and delivery of information. Total of 1631 patients were interviewed over telephone. It was concluded that proper management of perception and expectation of waiting time was the most effective approach to increase the satisfaction level of the patients.Forster et al. (2003) wrote a paper about Emergency Department (ED) overcrowding due to length of stay and disposition of patients. This study was conducted on acute care teaching centre of 500 bed capacity. The data were taken from two sources, Emergency Patients Information System (EPIS) and readings taken from each visit to ED. Data were collected on daily admission and consultation rate basis while considering length of stay of each patient from 1993 to 1999. The overall occupancy of hospital was found to be 89.7% with total of 155 patients coming to ED daily. From this number, 21% of patients were sent to physicians and 19% got admitted. On an average, a single patient spent 5 hours and 54 minutes in ED. When occupancy was increased by 10% then it influenced length of stay by increasing the latter number by 18 minutes. However, change in occupancy didn't affect consultation and admission rate and they remained unchanged. It was clearly evident that when length of stay increased then overcrowding of ED happened and this overcrowding could be dealt by increasing the number of beds.

Green et al. (2006) studied the variation of arrival of patients in emergency department (ED). The main focus of the study was on patients who leave ED without being seen. Queuing model was used to optimize the performance of ED by changing the pattern of staff. Total of 39 weeks

were used to evaluate the pattern of arrival of patients with the help of Lag SIPP Queuing Analysis model to simulate the strength of staff according to patients' arrival. Increasing the service hours of staff by twelve hours per week, helped to reduce the unattended number of patients by 258 patients. Despite the increase in arrival rate, the implementation of queuing theory helped to reduce the number of unattended patients by 161, by reallocation of the providers in the second part of the study. It was concluded that the timely management of the situation with the help of queuing theory could easily enhance the level of quality of service.Bruin et al. (2007) investigated the bottlenecks of emergency department in the specific area of cardiac in-patient flow. The main motto of the study is to find out the optimum number of beds required to minimize the adverse effect of refused admissions. First Cardiac Aid (FCA) was the first step to enter the procedure of getting admission in cardiac emergency unit. Failed admission to FCA was kindled mainly due to unavailability of beds. Furthermore, unstable values of length of stay and arrival rate made the work of staff more tedious. Queuing theory was used to decipher the complexity and dynamicity of the in-patient flow pattern.

Mayhew & Smith (2008) analysed UK government's claim of admitting patient within four hours of time in Accident and Emergency department with the help of queuing theory. A model is constructed to determine the main steps included in the treatment of patients coming to A&E department. Based on initial model, it was found that 90 percentile patients were treated within four hours of waiting time in 2003, while this statistic rose to 98 percentiles in 2007. Then initial model was modified by abolishing long treatment stage 2 and treating these patients outside the A&E department. The Re-design model came to be more effective than initial model and helped to clear congestion of patients in less time. It was concluded that giving more success percentile to walk in services say 98%, would be more beneficial and government's claim of reaching 100% of A&E in 4H time was found to be reached 98% from 90%, which was now a new standard time.Singer and Donoso (2008) described operational parameters and decision-making scenarios to provide ambulance facility. Key performance indicators for both patients and manager were assisted with help of queueing theory. This study took place in Chile. Authors divided ambulance facility in three sub processes and those were capturing the request, responding to the request and finishing the service. KPI for the manager was mainly the efficiency of resource utilization while it was a blend of three factors for patients and these were

diligence, kindliness and effort given. Further, six peak hours were taken under consideration from 18:00-24:00 and rate of calls were noted in a span of one-hour interval. Emergency and primary health care were included in this study. Then evaluation of present service was done and after applying queuing theory, the statistics of both were compared. It was interpreted that each KPI was contradicting another KPI, as mangers were more concerned about reducing the idleness without giving much importance to waiting time and customers were more interested in taking service as quickly as they could without considering the cost of providing the concerned service. Also, measuring effectiveness of the system could not done by hit and trial method as it was a matter of life and death. Although the main objective of the paper which was to show the application of queueing theory in managing ambulance service was fulfilled.

Cochrana and Roche (2009) concentrated on increasing the capacity of the emergency department by adopting a queuing model to create a proper network. All variation in arrival rate, service rate and other unstable parameters were amalgamated in a single computational model. The main objective of this paper was to eradicate the situation of 'walk-away'. Metrics of management were created for each area to contemplate its performance level in real time. The variation in patterns was noted on daily and monthly basis. As compared to computer computational model, spreadsheet approach was found to be more effective. The results obtained from this study were purposed to the management for discussion and their feedback was taken as well.Vass and Szabo (2015) analysed problems of emergency department by questioning 2195 patients who attended emergency department recently. The main motive of questionnaires was to increase the quality of the service. The study was conducted from 2010-2012. In this survey, 84.63% of participants were found to be satisfied with their overall experience. Most common complaints were about inadequate staff strength, small size of waiting room and long waiting times. This study helped to attain a strong relationship between all these problems and queue theory was used to make a decision making and optimal solution was adopted for the best performance of the emergency department.

Xie et al. (2016) studied the concept of reverse triage policy; where less wounded is given priority in giving a medical treatment. This concept is very useful in case of war scenarios where recourses are limited. In this study, a queuing model was developed with transfers and

abandonments by considering deterioration of condition of patients with the passage of time. To treat a non-critical patient, important conditions were derived with the help of smoothed rate truncation method. In conclusion, they suggested to adopt dynamic control policy; best suited according to a specific condition of a country and a hospital.

Hu et al. (2017) examined the importance of queuing theory to eradicate problems of emergency department. The study was conducted to make emergency department capable of mitigating the limitation of its applications by proper modelling of patients' behaviour. To make this study more efficient, different discrete methods were used to validate a single problem and accordingly their results were compared to find out the root causes of the problem. Queuing theory was used mainly on two major problems of demand-supply and complexity of emergency department by innovating new techniques. It was found that queuing theory generally over simplify problems and results obtained from the problems were less realistic. The use of combination of queuing theory with simulation technique was recommended for future studies. Yousefi et al. (2018) developed an agent-based modelling framework to get the idea about the pattern of patients who leave emergency department of the hospital without being seen by the doctors. Computer modelling and cellular automata techniques were used to fulfil the purpose of the study. A real case study was used to validate the result of the data and it was concluded that mainly four factors had influenced on the outcomes. Those were increasing the waiting room capacity for patients' convenience, practicing of fast-track treatments, increasing the number of nurses in triage section and reducing the time of actual treatment. After applying these preventing policies, 42.12% decrease in patient leaving without being seen was calculated. Also, length of stay decreased by 6.05%. The combination of queuing theory and cellular automata helped to achieve total of 11% improvement in the service area of emergency department.

In the span of 1996 to 2018, a number of papers were focused on ED and Ambulance departments of the different hospitals. These studies clearly show that queuing theory is a very useful approach to handle the congestion problem of emergency department. Ambulance service was also enhanced with the help of queuing theory.

### 4.2. Intensive Care Unit (ICU)& Out-Patient Department (OPD)

Intensive Care Unit is a special unit that provides special intensive medicine treatment to the patients. ICU mainly deals with the critical life-threatening conditions of the patients. Diseases are diagnosed by highly trained doctors and staff. On the other hand, Out Patient Department deals with patients seeking minor treatments. Patients in OPD don't require to stay overnight and diagnoses are done on the same day.

Kim et al. (1999) analysed the processes of admission and discharge of patients from intensive care unit (ICU) in a public hospital of Hong Kong. The queuing and computer simulation models were used to analysed the actual six months data of ICU. It was found earlier that the decision-making scenarios of ICU were completely subjective and this behaviour initiated the need of this study to give specific criteria on which decisions could be made. The arrival of patients who came to ICU was from four sources Ward, A&E, OT-Emergency and OT-Electives. Patient's welfare status was found to be most dominant factor followed by available scarce resources of the unit and availability of beds. Based on arrival rate of the patients, 14 ICU beds were sufficient to use the facility at optimal performance.Gallivan et al. (2002) conducted study at St George's Hospital Medical School, London on 7014 patients who entered intensive care after cardiac surgery. He tried to relate booking of medical facility with a flight facility in advance. He concluded that even the average length of stay of patients in intensive care was around 1.65 days but still there were some substantial variations present. These variations couldn't be neglected when producing a booking system for the surgical operations. On an average, eight to nine beds sufficed the need of intensive care unit to operate efficiently. The length of stay is the most important factor to determine effective hospital operation and simple tasks like booking flight tickets would be not prudent to compare with booking hospital service in advance.

Joustra et al. (2010) studied on a decision of 'to pool or not to pool' in a radiotherapy outpatient department of hospital. Queuing theory and discrete event simulation were used to make this case study more realistic. The practical approach of this study showed that separate lines would be more efficient if they are categorized as regular and urgent patients line. The results gave practicality of this approach in healthcare centres. Also, this approach was approved by theoretical concept of queuing theory. It was concluded that dividing service priorities towards

different categories of customers would be beneficial if the relative processing time difference is significant.Bahadori et al. (2014) aimed to make the management of outpatient pharmacy more efficient with the help of suitable queuing theory and simulation technique. This study was done at military hospital in Iran, Tehran in 2013. A total of 220 patients were studied to determine arrival rate, service rate and other data to calculate performance variables of queueing network. This study was done on two shifts i.e. morning and evening. To set up an initial network of the situation, SPSS 18 software was used. After that, an appropriate solution was made with the help of ARENA 15 & 4 software. Results of this study showed that the queue network of the pharmacy was unpleasant in morning and as well in evening shifts. The average numbers of patients were 19.21 and 14.66, and average times spent in the system by patients were 39 and 35 minutes in the morning and evening, respectively. The utilization of the system was 25% and 21% in the morning and evening. The simulation results concluded that mitigating staff in the morning from two to one in the receiving prescriptions stage didn't change the performance of the queue indicators but allotting one extra member in filling prescription drugs could decrease ten persons in the queue length and approximately 18 minutes in the waiting time. On the flip side, in the evening, reducing the staff member from two to one in prescription drugs delivery personnel, changed the queue performance indicators in very small manner. Increasing a strength of staff members to fill prescription drugs could decrease 5 persons in queue length and approximately nine minutes in the average waiting time. Overall time taken could be reduced by using multitasking persons and managing staff to the time-consuming stage of filling prescriptions, with the help of suitable usage of queuing theory and simulation techniques.

Mustafa and Nisa (2015) studied the different departments of hospital to find a best suit to follow single or multi server queuing theory. All the queuing problems were assumed to follow exponential distribution in arrival of patients and Poisson distribution in serving these patients. Both M/M/1 and M/M/2 were implemented on queues to find remaining queue parameters. The general queue simulation was run on these parameters to analyse the network of queuing theory. Apart from this, various parameters were compared and altered to find their relationship and effectiveness on the queuing system.Dilrukshi et al. (2016) studied the pattern of patients coming to the national hospital of Sri Lanka. They tried to get an optimum result of hospital's staff to attend maximum number of patients in the minimum time. According to the pattern and number

of the patients coming in a specified time, different counters were made open. Maximum counters were made open when the numbers of patients were maximum. This study helped to know the peak hours of rush and accordingly allocation of adequate staff strength to handle congestion of patients.

Saddiqui et al. (2017) studied the required staff strength of the intensive care unit to satisfy patients coming to the department. The main purpose of this study was to estimate the strength requirement of staff to serve patients better. It was found that current approaches were not reliable to give right estimation of staff strength. Discrete event simulation was coupled with queuing theory to get the estimation of staff required to handle department congestion problems without facing any major issue. Dynamic approach of assigning nurses to patients was implemented after finding out patients to nurse ratio factor. It was concluded that amalgamation of queuing theory with simulation technique is more beneficial as compared to using queuing theory alone. Bai et al. (2018) reviewed studies carried out on intensive care unit by using queuing theory. The main purpose of the research was to perceive the importance of operation research and quality management in handling problems related to healthcare sectors. Different problems were tackled and sorted out ranging from decision horizons, setting of different situations according to demand and supply policy. Apart from that, gaps in studies and opportunities for future result were notified.

From 1999 to 2018, queuing theory was used to make the overall system of ICU and OPD smooth. Various problems of queues were tackled with precision with the help of queuing theory.

### 4.3. Others Departments

Fetter and Thompson (1966) studied the acceptable waiting time to the patients and tried to balance the patients' waiting time and the doctors' idle time. They concluded that there are at least seven variables affecting this relationship: appointment interval, service time, patients' arrival pattern, number of no-shows, number of walk-ins, physicians' arrival pattern, and interruptions in patient services. They suggested that taking all variables into account is not possible and considering one variable and its effects is a better approach to get the idea about the

effectiveness of that parameter. Iversen (1993) wrote an article by considering his own third technique of queuing theory. In the first approach, arrival and service distributions were taken as stochastic events and second approach emphasized on individual behaviour of the subjects. The third approach was related to waiting time of hospitals and it was derived from intuitional theory of waiting time. This article gave a direct relationship between revenue of hospital and their waiting time list. In some situations, to obtain an optimal budget, adopting long waits theory was more beneficial. But this theory was detrimental to other departments like varicose veins, hernias and vasectomies, where it was negatively influenced by long waits and hospitals lost their revenues. In General, the less waiting time meant that hospital was more attractive to patients.
Iversen (1997) examined the effect of a private sector on the waiting time associated with treatment in a public hospital. He concluded that without rationing of waiting-list admissions, a private sector was shown to result in a longer waiting time if the demand for a public treatment was sufficiently elastic with respect to the waiting time. When waiting-list admissions were rationed, the waiting time was shown to increase if the public-sector consultants were permitted to work in the private sector in their spare time.Pruyn and Smidts (1998) studied the effects of objective waiting time and waiting environment on satisfaction with the service, were investigated. Two elements of the waiting environment were distinguished: the attractiveness of the waiting room and the presence of television TV. as an explicit distracter. The adverse effects of waiting can be soothed more effectively by improving the attractiveness of the waiting environment than by shortening the objective waiting time. Objective waiting time influences satisfaction mainly via a cognitive route: through perceived waiting time in minutes. and the long short judgment of the wait.

Fomundam and Herrmann (2007) surveyed the contributions and applications of queuing theory in the field of healthcare. The paper summarized a range of queuing theory results in the following areas: waiting time and utilization analysis, system design, and appointment systems. The paper also considered results for systems at different scales, including individual departments (or units), healthcare facilities, and regional healthcare systems. The goal was to provide sufficient information to analysts that who are interested in using queuing theory to model a healthcare process and want to locate the details of relevant models. After surveying all the aspects, they concluded that Specifying utilization factor only leaded to congestion, fixing

patient to resource ratio would lead to poor service quality and appointment systems were designed to avoid doctor's idle time without considering patient's waiting time.Zonderlandet al. (2009) studied an amalgamation of mathematical techniques and staff strength that supported a better decision-making system. The study was conducted on paranaesthesia evaluation clinic of a university hospital where admission was done by either walk-ins or appointments. The data was collected from logging system of clinic and by observation about arrival and service rate. Two parameters were focused and these parameters were length of stay and employees' utilization. An inceptive queuing model was suggested to modify for better service. Total of three models were having more effectiveness than inceptive model and management was suggested to adopt one of these models to run the clinic in more effective manner.

Li et al. (2009) tried to implement a model which is based on combination queuing theory and goal programming, to make a multi-objective decision making system for Zichuan hospital, China. This hospital had 11 departments and bed allocation decision making was dealt in this paper. This research paper took into account some previous researches to find a better solution. Queuing theory was used to estimate the arrival of patients and model M/PH/m was used with arrival pattern to be Poisson in nature. Further GP was used to compare stats for hospital's profit and success rate of serving patients. Mainly, Targets for GP were set considering two parameters i.e. patient admission (%) and profit per day. The overall value of patient admission and profit of the hospital increased with the help of this model.Obamiro (2010) studied the time taken by patients (except mothers) to get served in a hospital. He observed the extent of effectiveness of queuing theory in ante-natal unit of a public teaching hospital in Nigeria. Tora optimization system was used to analyse data of a hospital to find out the effective parameters of the ante-natal unit. The data of this study was collected for over three weeks period. It was shown that pregnant ladies were given priority in first week of arrival as compared to next two succeeding weeks and they spent less time in queue in their first week of treatment. This study implied that there are lesser numbers of women in the system in first week.

Mehandiratta (2011) studied an Operational research's approach to handle problems of healthcare sectors. The main problem was the congestion of patients to get served in a hospital. This problem was suggested to deal with a queuing theory. She concluded that the Queuing

theory has been used in all other sectors but its use in healthcare sectors is lagging behind other sectors. Queuing theory has use in every domain of hospital like inpatient, outpatient, emergency preparedness, pharmacy, inventory control etc. Advancement of technical approach and pressure of providing high quality service are pushing authorities to use queuing theory to provide better and safe environment for patients.Kumar and Sharma (2012) studied the problems related to reneging decision of customers.This decision making was considered as a main reason of losing potential customers.A new model was designed to counter this issue and customers were convinced to stay in a line by different mechanisms. A simple queuing model was implemented on the queue problem with arrival and service times to follow negative- exponential distribution. This model introduced a new probability parameter which gave idea of customers retaining in a queue. This customer retention variable was found to be directly proportion to the average size of the system.

Ameh et al. (2013) conducted a survey to reduce the waiting time and enhance the satisfaction level of patients. Total of 210 patients were given questionnaires about the time spent by them in queue, in doctor's office and how to improve the quality of the service. Among these 210 patients, 83 were men and 137 women aged 4-70 years old. 164 patients were found to spend two or less hours in queue and less than one hour with a doctor. 144 patients felt satisfied with their overall experience in a hospital. 46 patients suggested that more doctors should be employed to elevate the quality of the service. Apart from this, strictly obeying 'First Come First Serve' policy and doctors to be more punctual were other suggested measures.Takagi et al. (2014) studied the provided data of two years by Medical Information Department, University of Tsukuba Hospital in Japan. Obstetric patients were focused in this study and an attempt was made to decode the random patients flow of this unit. Little's law was used to confirm the flow of each patient to two different wards. First ward was for serious patients and second was for normal patients. The number patients staying in two wards were estimated by two models of queuing models. From these models, length of stay of patients was reckoned as well. This generated model was useful to propose the capacity of the obstetric unit if the arrival strength is known.

Komashie et al. (2015) investigated the interconnection between patients' satisfaction, time of waiting, time of service and satisfaction of the staff members. Both experiment and operational strategies were amalgamated together to sort out the problems related to queues and customers satisfaction. To get the measure of customers' satisfaction, waiting time parameter was used. Similarly, staff satisfaction was estimated from the time spent by staff with patients. These two models were combined together to create an effective satisfaction level model. Bahadori et al. (2017) studied the magnetic resonance imaging (MRI) department of hospital in Iran. The study emphasized on optimizing the performance of MRI department by using simulation and queuing theory techniques to increase the productivity of the department and satisfaction level of patients. The data of both day and night shifts were taken into account of 264 patients by quota sampling. Excel 2013 and Arena 14.5 were used to produce the result. The result showed that MRI department had huge potential but it had been wasted because of bad management techniques. Proper human resource management helped to increase the capacity of the system and it drastically decreased the waiting time of patients and hence increased satisfaction level of the patients.

Botani and Hassan (2018) studied the pattern of queuing network of CT scan unit in Erbil teaching hospital in a quest to solve problems related to restriction of line and mitigate the issue of long waiting time. EasyFit software V.5.4 with TORA software V.2 was used to analyse 30 days data of CT scan unit. This study was conducted to find the suitable queuing model for the unit and run it on optimal level of performance. It was concluded in the end of study that two server models helped to reduce the waiting time by 1.44 hours and management should practice this model in CT scan unit. Bittencourt et al. (2018) used queuing theory to manage the capacity of hospital and deal with the congestion problem due to increased demand of healthcare facilities. Queuing model was constructed on six months data of in-patients unit of university hospital. Implementation of queuing theory helped to find a bottleneck in management of bed capacity of hospital.

Other departments like Ante-Natal department, CT scan clinic and other departments also contributed to untap the potential of queuing theory. Problems of bed allocation and other resources allocation were extensively done with the help of queuing theory.

## 5. Statistical analysis of studies

As indicated earlier, total 35Nos. of research papers are used for this compilation.The statistical analysis is used to find trends and relationships between different studies at different time periods. The bar chart and pie chart have been used to analyse statistically. Analysis of studies of this paper is categorised into following sub-sections.

### 5.1. Analysis of Studies using Bar Chart

The bar chart is used to demonstrate the number of studies done in different periods. There were total of six studies done before 2000 on implementation of queuing theory in healthcare sectors. After that, this number reduced to two in a span of 2000-2004. A rise in trend was found in next period of 2005-2009 and number went to eight studies which is the highest value in this bar chart. Moving forward to next period, 2010-2014, seven studies were conducted on hospital in different department by using queuing theory. Onwards 2015, Twelve Nos. of studies have done so far and more studies are expected in coming years

### 5.2. Analysis of Studies using Bar Chart

The bar chart is used to demonstrate the number of studies done in different periods. There were total of six studies done before 2000 on implementation of queuing theory in healthcare sectors. After that, this number reduced to two in a span of 2000-2004. A rise in trend was found in next period of 2005-2009 and number went to eight studies which is the second highest value in this bar chart. Moving forward to next period, 2010-2014, seven studies were conducted on hospital in different department by using queuing theory. Onwards 2015, 12 studies have done so far and more studies are expected in coming years. Figure 3 shows the bar chart for distribution of 35 studies carried out in different time periods.

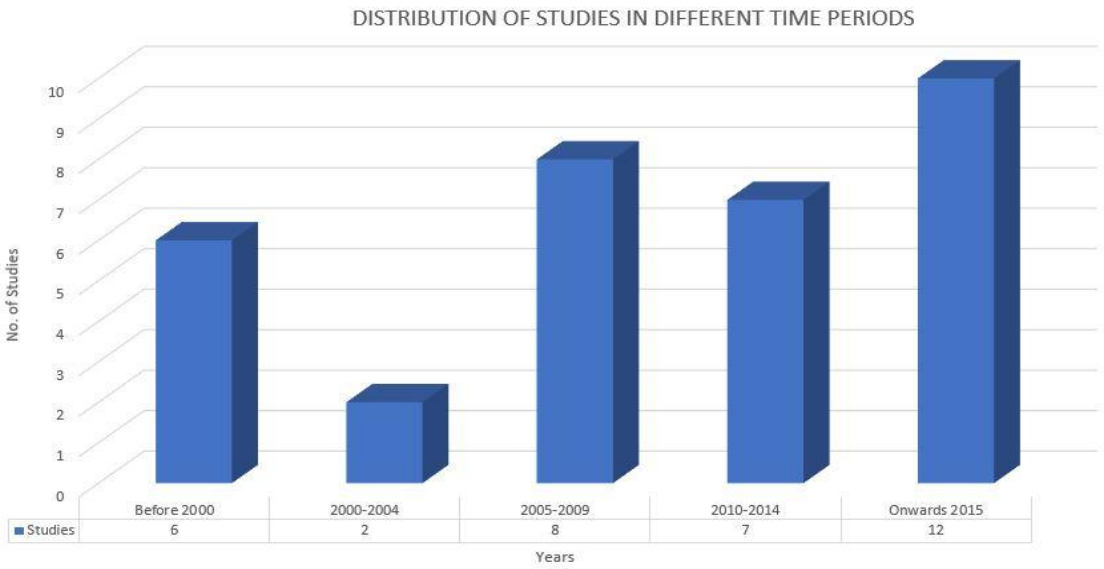


**Figure 3 Distribution of studies in different time periods**

### 5.3. Analysis of Studies using Pie Chart

Figure 4 shows the pie chart of studies carried out in different departments of hospitals from 1996 to 2018.

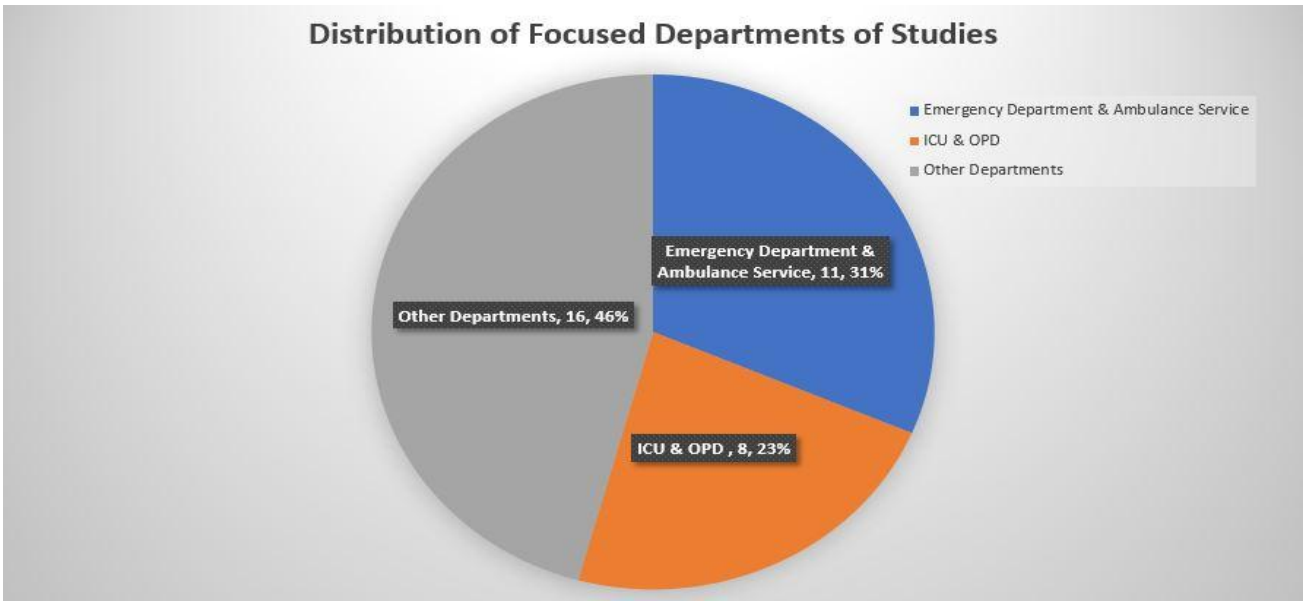


**Figure 4 Pie chart for distribution of studies on various departments**

The pie chart illustrates proportion per hundred of different departments that were used to implement queuing theory. Total of 11studies have been focused on emergency department and ambulance services. After that, eight studies were focused on Intensive care units(ICU) and outpatient departments (OPD) to increase the efficiency of these departments of the health care system. Remaining 16Nos. of studies have been performed on the other department like; Ante-Natal units, CT Scan units and other general departments.

## 6. Discussion

It is clear from the Figure 3 that the trend of using queue theory before 2000 was not very popular. Before 2000, queuing theory was taken as a tool to treat problems present in the industries. But this trend changed after 2000. Although, only two studies were conducted till 2004, this initiated a trend of using queuing theory in healthcare sectors. In the period of 2005-2009, eight studies have been done using queuing theory as a tool to mitigate the problems of healthcare departments. After that the trend remained stable with a hike in recent years and it is expected that it will increase even more with time and in coming years.It is expected that more Nos. of studies will be conducted in various departments of healthcare sector in near future.

Figure 4 clearly shows the trend of applications of queuing theory on different departments of hospitals. It is evident that 46% studies were done on other departments and 31% studies were focused on Emergency and Ambulance services.While remaining 23% studies had their prime focus on Intensive Care Unit (ICU) & Out-Patient Department (OPD).

## 7. Conclusion

Quality of healthcare sectors is the major concern of the hospitals because it directly influences their revenues. But longer waiting times and mismanagement of available resources are mitigating customers' satisfaction. This paper takes into account all concerns and shows the potential use of queuing theory in healthcare sectors.

This review paper has amassed the knowledge of queuing theory applied in healthcare sector during the span of 1996 to 2018. There are number of studies those have been conducted before 1996 and have significant role in improving the standard of quality level. Most of the studies included in this review paper has a common limitation of assuming an ideal model of process and authors failed to execute models based on real life conditions.

The literature survey of this paper shows that how a simple implementation of basic models of queuing theory can be used to handle majority of problems related to overcrowding and even distribution of available resources in a proper manner. All departments of hospitals have a large scope for queuing theory. Due to ease of queuing theory and its positive results, trend of using it has been increased drastically from 2000. Despite of increased trend of queuing theory in hospitals in recent years, it still has a huge potential to tackle bottlenecks of healthcare sectors and it will help generously to increase the standards of quality in coming years.

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