International Journal of Management, IT & Engineering
Vol. 8 Issue 11, November 2018,
ISSN: 2249-0558 Impact Factor: 7.119
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

COUPLED ADDITION MODULO M-DIMENSIONAL GROUPS

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Abstract: We introduce the concept of coupled additive modulo m-dimensional group with respect to T-norm. Using this we analyze the characterizations of coupled additive modulo m-subgroup with suitable examples.

Keywords: Fuzzy set, t-norm, coupled fuzzy set, m-fuzzy subgroup, additive m-group, generalization, membership function, ƛ-cutset.

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Introduction: Rosenfeld [6] defined the concept of fuzzy group and explained basic results.In 1965 Zadeh [7] mathematically formulated the fuzzy subset concept. He defined fuzzy subset of a non-empty set as a collection of objects with grade of membership function in a continuum, with each object being assigned a value between 0 and 1 by a membership function. A note on ordered semigroups was introduced by [2]. H.Aktas [3] investigated the generalization of fuzzy subgroups with respect to t-norms.Fuzzy set theory [4] was guided by the assumption that classical sets were not natural, appropriate or useful notions in describing the real life problems, because every object encountered in this real physical world carries some degree of fuzziness. Subgroups and its similarities has been encounter by [5].Further the concept of grade of membership is not a probabilistic concept. A new type of fuzzy normal subgroups and fuzzy cossetshas been discussed in [1]. In this article,we introduce the concept of coupled additive modulo m-dimensional group based on a mathematical tool T-norm. Using this we analyze the characterizations of coupled additive modulo m- subgroup with suitable examples.

Section-2 Preliminaries

We record here some basic concepts and clarify notions used in the sequel.

Definition-2.1: Let X be a non-empty set. A mapping A: X→ [0,1] is called a fuzzy set in X.

Example 2.2: Let X = {a,b, c,d,e}. Then the fuzzy set A in X is defined by

A= {a/0.2 , b/0.8 , c/0.2 / d/0.5, e/0.4}.

Definition -2.3: A fuzzy subset A of a group (G , ⊕) is said to be a coupled addition modulo m-dimensional subgroup of G if for all x,y ε G,

(CMG1) $A(x \oplus y)^m \geq \inf \{A(x^m), A(y^m)\}$

(CMG2) $A(-x)^m = A(x^m)$, where ⊕ is the addition modulo of x andy is denoted by x ⊕y and the inverse of x by –x.

Example-2.4: Let (Z_4, k) be a 4- ary group derived from the additive group Z_4. It is not difficult to see that the map A defined by A(0) = 0.6 and A(x) = 0.7 for all x ≠ 0 is a coupled additive modulo 4-ary subgroup in which for x = 2 we have negative of x is zero and A(-x) > A(x).

Definition-2.5: A triangular norm (briefly a t-norm) is a function T: [0,1]× [0,1]→ [0,1] satisfying for each (x,y,z,p) ε [0,1],

T1: T(x ,1) = x

T2: $T(x,y) \leq T(z,p)$ if $x \leq z$ and $y \leq p$.

T3: $T(x,y) = T(y,x)$

T4: $T(x, T(y,z)) = T(T(x,y), z)$.

Definition-2.6: Let G be a coupled addition modulo m-groupoid and T a t- norm

A function B: G → [0,1] is a subgroupoid of G if for every x, y ε G,

$B(x \oplus y)^m \geq T(B(x^m), B(y^m))$.

Definition-2.7: For each i = 1,2,3 ………., n , let A_i be a coupled addition modulo m- fuzzy sets X_i. $\alpha_1,\alpha_2,\ldots\ldots\ldots, \alpha_n$ ε Ꝁ , then

(1) $\alpha_1 A_1 \oplus \alpha_2 A_2 \oplus \ldots\ldots\ldots \oplus \alpha_n A_n \subset A$

(2) $A(\alpha_1 x_1 \oplus \alpha_2 x_2 \oplus \ldots\ldots\ldots \oplus \alpha_n x_n)^m \geq \inf\{A_1(x_1{}^m) \oplus A_2(x_2{}^m) \oplus \ldots\ldots\ldots \oplus A_n(x_n{}^m)\}$ for all x_1, x_2, …. x_n ε X.

Definition-2.8: Let Φ : X→ Y be a function for a coupled addition modulo m-fuzzy set A in Y, we define $\Phi^{-1}((A)(x))^m = A(\Phi(x^m))$ for every x ε X.

For a coupled addition modulo m-fuzzy set β in X , Φ(β) is defined by

$$(\Phi(\beta))^m(y) = \begin{cases} \sup \beta(x^m) \text{ if } \Phi(z) = y \text{ , } z \text{ ε } X \\ = 0 \quad \text{otherwise.} \end{cases}$$

Definition-2.9: Let $A_1, A_2, \ldots\ldots\ldots A_n$ be a coupled addition modulo m-fuzzy sets of X, define

(i)$\Phi(A) = A_1 \oplus A_2 \oplus \ldots\ldots\ldots A_n$, where $\Phi(x_1,x_2,\ldots.. x_n)^m = x_1{}^m \oplus x_2{}^m \ldots.. \oplus x_n{}^m$

(ii) $A(x_1, x_2,\ldots\ldots x_n) = \min\{A_1(x_1{}^m), A(x_2{}^m)\ldots\ldots A(x_n{}^m)\}$ for all x ε X.

3. PROPERTIES OF COUPLED ADDITION MODULO M-FUZZY SUBSET

Theorem-3.1: Let $A_1, A_2, \ldots..,A_n$ be a coupled addition modulo m-fuzzy subsets of the sets G_1, $G_2, \ldots\ldots,G_n$ respectively.Then

$(A_1 \oplus A_2 \oplus \oplus A_n)^m(Z) = \sup\{\min\{A_1(x_1{}^m), A_2(x_2{}^m),\ldots\ldots.,A(x_n{}^m)\}$

$Z= x_1 \oplus x_2 \oplus \ldots\ldots. \oplus x_n$

Proof: Since $\Phi(A) = A_1 \oplus A_2 \oplus \ldots\ldots\ldots. \oplus A_n$, using definition-2.9 and definition-2.8

$(A_1 \oplus A_2 \oplus \ldots\ldots\ldots \oplus A_n)^m (Z) = \sup A(x)$, $x = (x_1,x_2,\ldots\ldots.,x_n)$ ε G^m

$\Phi(x)= Z\Phi(x) =Z$

Since $\Phi(x)= \Phi(x_1,x_2\ldots\ldots.x_n) = x_1 \oplus x_2 \oplus \ldots.. \oplus x_n$, using definition -2.9

$A(x) = A(x_1,x_2,\ldots\ldots,x_n) = \min\{A_1(x_1), A_2(x_2), \ldots\ldots A_n(x_n)\}$

Then $(A_1 \oplus A_2 \oplus \ldots\ldots. \oplus A_n)(Z) = \sup\{\min\{A_1(x_1), A_2(x_2), \ldots\ldots A_n(x_n)\}$

$x_1 \oplus x_2 \oplus \ldots\ldots \oplus x_n$.

Theorem-3.2: Let $A_1, A_2, \ldots. A_n$ be a coupled addition modulo m-groups $G_1, G_2, \ldots.. G_n$ respectively. Then $(A_1 \oplus A_2 \oplus \ldots\ldots. \oplus A_n)$ is coupled addition modulo group of $G_1, G_2, \ldots\ldots. G_n$.

Proof: we must show that $((A_1 \oplus A_2 \oplus \ldots\ldots. \oplus A_n)$ is a coupled addition modulo subgroups of $G_1, G_2, \ldots\ldots. G_n$.

We get

$((A_1 \oplus A_2 \oplus \ldots\ldots. \oplus A_n)^m (x_1, x_2 \ldots.. x_n) \oplus (y_1, y_2, \ldots. y_n)$

$= (A_1 \oplus A_2 \oplus \ldots\ldots. \oplus A_n) (x_1 \oplus y_1, x_2 \oplus y_2, \ldots\ldots. x_n \oplus y_n)$.

Let $(x_1 \oplus y_1, x_2 \oplus y_2, \ldots\ldots. x_n \oplus y_n) = Z$

$= \sup \inf \{ A_1(x_1 \oplus y_1), A_2 ((x_2 \oplus y_2), \ldots\ldots\ldots\ldots A_n((x_n \oplus y_n)\}$

$(x_1 \oplus y_1, x_2 \oplus y_2, \ldots\ldots. x_n \oplus y_n) = Z$

$\geq \sup \inf \{\inf (A_1(x_1), A_2(x_2), \ldots\ldots. A_n(x_n), \inf (A_1(y_1), A_2(y_2), \ldots\ldots A_n(y_n)\}$

$(x_1 \oplus y_1, x_2 \oplus y_2, \ldots\ldots. x_n \oplus y_n) = Z$

$= \min \{(A_1 \oplus A_2 \oplus \ldots\ldots\ldots. A_n) (x_1, x_2, \ldots. x_n), \{(A_1 \oplus A_2 \oplus \ldots\ldots\ldots. A_n) (x_1, x_2, \ldots. x_n)\}$

Also

$(x_1 \oplus y_1, x_2 \oplus y_2, \ldots\ldots. x_n \oplus y_n) = Z = (x_1 \oplus y_1, x_2 \oplus y_2, \ldots\ldots. x_n \oplus y_n) = Z$

$= \sup \inf \{(A_1(x_1), A_2(x_2), \ldots\ldots. A_n(x_n)\}$

$(x_1 \oplus y_1, x_2 \oplus y_2, \ldots\ldots. x_n \oplus y_n) = Z$

Since $((A_1 \oplus A_2 \oplus \ldots\ldots. \oplus A_n)^m$ is coupled addition modulo m-groups of G_i

$(x_1 \oplus y_1, x_2 \oplus y_2, \ldots\ldots. x_n \oplus y_n) = Z$

$= \sup \inf \{ A_1(-x_1), A_2 ((-x_2), \ldots\ldots\ldots\ldots A_n(-x_n)\}$

$(x_1 \oplus y_1, x_2 \oplus y_2, \ldots\ldots. x_n \oplus y_n) = Z$

$= ((A_1 \oplus A_2 \oplus \ldots\ldots. \oplus A_n)^m (-x_1, -x_2, \ldots\ldots., -x_n)$

$= ((A_1 \oplus A_2 \oplus \ldots\ldots. \oplus A_n)^m (- (x_1, x_2, \ldots\ldots.., x_n))$

Thus

$((A_1 \oplus A_2 \oplus \ldots\ldots.. \oplus A_n)^m$ is coupled addition modulo m-group of Gi.

From the following section, we introduce the definition of level cut of coupled addition modulo m-subgroup.

Definition-3.3: Let A be a coupled addition modulo m-fuzzy subset of a set G, T a t=norm and $\lambda \varepsilon [0,1]$, then we define λ-cut subset of a coupled addition modulo m-fuzzy subset of A as $A_\lambda^T = \{x \varepsilon G / \sup T(A(x^m), \lambda) \geq \lambda \}$

x εG

Theorem 3.4: Let G be group and A a λ-coupled addition modulo m- fuzzy subgroup of (G , ⊕) then a λ-cut subset $A_\lambda^T = \{x \varepsilon G / \sup T(A(x^m), \lambda) \geq \lambda \}$ is λ-coupled addition modulom-fuzzy subgroup of (G , ⊕) where e is the identity of G.

Proof:

Let x,y ε A_λ^T . Then $\sup T(A(x^m), \lambda) \geq \lambda$ and $\sup T(A(y^m), \lambda) \geq \lambda$, since A is λ-coupled addition modulo fuzzy subgroup of (G , ⊕), then $A(x \oplus y)^m \geq T(A(x^m), A(y^m)$ is satisfied. This means

$\sup T(A(x \oplus y)^m, \lambda) \geq \sup T(T(A(x^m), A(y^m), \lambda)$

$\sup T(A(x^m), \lambda) \geq \lambda$ or $\sup T(A(y^m), \lambda) \geq \sup T(A(x^m), \lambda) \geq \lambda$ and $\sup T(A(y^m), \lambda) \geq \lambda$

hence $x \oplus y \varepsilon A_\lambda^T = \{x \varepsilon G / \sup T(A(x^m), \lambda) \geq \lambda \}$.

Again x ε $A_\lambda^T = \{x \varepsilon G / \sup T(A(x^m), \lambda) \geq \lambda \}$ implies $\sup T(A(x^m), \lambda) \geq \lambda$

Since A is a λ-coupled addition modulo m-fuzzy subgroup of (G , ⊕) ,$A(-x^m) = A(x^m)$ and hence $\sup T(A(-x^m), \lambda) = \sup T(A(x^m), \lambda) \geq \lambda$ this means that $-x \varepsilon A_\lambda^T = \{x \varepsilon G / \sup T(A(x^m), \lambda) \geq \lambda \}$.

Theorem 3.5: Let G be group and A a λ-coupled addition modulo m-fuzzy subgroup of (G , ⊕), then a λ-cut subset $A_\lambda^T = \{x \varepsilon G / \sup T(A(x^m), \lambda) \geq \lambda \}$. For $\lambda \varepsilon [0,1]$ is a subgroup of (G , ⊕) where e is the identity of G.

Proof:

Let x,y ε A_λ^T . Then $\sup T(A(x^m), \lambda) \geq \lambda$ and $\sup T(A(y^m), \lambda) \geq \lambda$, since A is λ-coupled addition modulo fuzzy subgroup of (G , ⊕), then $A(x \oplus y)^m \geq T(A(x^m), A(y^m)$ is satisfied. This means sup $T(A(x \oplus y)^m, \lambda) \geq$ sup $T(\inf (A(x^m), A(y^m), \lambda)$, where there are two cases

$\inf(A(x^m), A(y^m)) = A(x^m)$ or $\inf(A(x^m), A(y^m)) = A(y)$ since x, y $A_\lambda^T = \{x \in G / \sup T(A(x^m), \lambda)$

Also in to case sup $T(\inf (A(x^m), A(y^m), \lambda) \geq \lambda$

Therefore sup $T(A(x \oplus y)^m, \lambda) \geq \lambda$, thus we get $x \oplus y \in A_\lambda^T = \{x \in G / \sup T(A(x^m), \lambda) \geq \lambda\}$.

It is easily seen that, as above $-x \in A_\lambda^T = \{x \in G / \sup T(A(x^m), \lambda) \geq \lambda\}$.

Hence $A_\lambda^T = \{x \in G / \sup T(A(x^m), \lambda) \geq \lambda\}$ is a subgroup of G.

Theorem 3.6: Let A and B be λ- cut subset of the sets G and H respectively, and $\lambda \in [0,1]$,then $A \oplus B$ is also a λ- cut subset of $G \oplus H$.

Proof: Since any t-norm T is associative, using definition 2.5 and definition-3.3 we can write, the following statements

sup $T((A\oplus B)(x \oplus y)^m, \lambda) \geq$ sup $T(\sup \inf (A(x^m), B(y^m), \lambda)$,

$= \sup T(A(x^m))$, $\sup T(\inf (B(y^m), \lambda)$

$\geq \sup T(A(x^m), \lambda) = \lambda$.

CONCLUSION:H.Aktas [2006] investigated the generalization of fuzzy subgroups with respect to t-norms. Fuzzy set theory [1980] was guided by the assumption that classical sets were not natural, appropriate or useful notions in describing the real life problems, because every object encountered in this real physical world carries some degree of fuzziness. we have discussed the concept of Coupled addition modulo fuzzy subgroups and suitable examples are given. Also , we investigate the properties of coupled addition modulo m-fuzzy sets in terms of groups with respect to suitable norm.

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