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# ADVANCED INTUITIONISTIC FUZZY OPERATORS AND ITS PROPERTIES

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**Abstract**

Different operations on intuitionistic fuzzy sets (IFSs) are defined in the existing literature and several operators are introduced over IFSs in a series of research. In this paper some new type of operators ($\cup_2$, $\cap_2$, $+_2$, $\cdot_2$, $@_2$, $\$_2$, $\#_2$, $*_2$) are proposed which are analogous to the existing operators $\cup$, $\cap$, $+$, $\cdot$, $@$, $\$$, $\#$ and $*$. Some new equalities connected with the proposed intuitionistic fuzzy operators are proved.

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## 1. Introduction

Intuitionistic fuzzy sets(IFSs) as a generalisation of fuzzy sets[11] was introduced by Atanassov.K[1]. At first the intuitionistic fuzzy operator was defined by Atanassov.K[3]. Several operators are defined in intuitionistic fuzzy set theory and properties of the operators were discussed by many researchers. In [9,10] , R.K.Verma and B.D.Sharma proved new equalities associated with the operations $\rightarrow, \cup, \cap, +, \cdot, @, \$, \#$ and $*$. In [8] Vasilev.T framed four equalities by using the operators $+, \cdot, @, \$, \cap$ and $\cup$.

In this present context we defined some of the operators as an extension of operators defined by R.K.Verma and B.D.Sharma[8,9].The aim of this paper is to obtain new equalities related to the proposed operators.

This paper is organised as follows: In section 2, some basic definitions related to IFSs are presented. In section 3, new intuitionistic fuzzy operators are defined and some results related to the proposed operators are proved.

## 2.Preliminaries

### 2.1 Definition[2]

An intuitionistic fuzzy set defined on a universe of discourse X is mathematically represented as $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ where the functions $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ to the set A respectively for every $x \in X$ such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. Further $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called hesitancy degree of $x$ in A.

### 2.2 Definition [2]

Let IFS(X) denote the family of all Intuitionistic Fuzzy Sets in the universe X. Assume A, B $\in$ IFS(X) given as $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$, $B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\}$. Some set theoretic and arithmetic operations are defined as follows:

i) $A^c = \{< x, \nu_A(x), \mu_A(x) > | x \in X\}$

ii) $A \cup B = \{< x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) > | x \in X\}$

iii) $A \cap B = \{< x,\ min(\mu_A(x), \mu_B(x)),\ \max(\nu_A(x), \nu_B(x)) > | x \in X\}$

iv) $A \cdot B = \{< x,\ \mu_A(x)\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x) > | x \in X\}$

v) $A + B = \{< x,\ \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x),\ \nu_A(x)\nu_B(x) > | x \in X\}$

vi) $A \mathbin{@} B = \{< x,\ \frac{\mu_A(x) + \mu_B(x)}{2},\ \frac{\nu_A(x) + \nu_B(x)}{2} > | x \in X\}$

vii) $A \$ B = \{< x, \sqrt{\mu_A(x)\mu_B(x)}, \sqrt{\nu_A(x)\nu_B(x)} > | x \in X\}$

viii) $A \# B = \{< x,\ \frac{2\mu_A(x)\mu_B(x)}{\mu_A(x) + \mu_B(x)},\ \frac{2\nu_A(x)\nu_B(x)}{\nu_A(x) + \nu_B(x)} > | x \in X\}$ for which we shall accept that

if $\mu_A(x) = \mu_B(x) = 0$, then $\frac{\mu_A(x)\mu_B(x)}{\mu_A(x) + \mu_B(x)} = 0$ and if $\nu_A(x) = \nu_B(x) = 0$

then $\frac{\nu_A(x)\nu_B(x)}{\nu_A(x) + \nu_B(x)} = 0$

ix) $A * B = \{< x,\ \frac{\mu_A(x) + \mu_B(x)}{2(\mu_A(x)\mu_B(x) + 1)}, \frac{\nu_A(x) + \nu_B(x)}{2(\nu_A(x)\nu_B(x) + 1)} > | x \in X\}$

x) $A \to B = \{< x, \min(\nu_A(x), \mu_B(x)), \max(\mu_A(x), \nu_B(x)) > | x \in X\}$

**2.3 Definition [2]**

Let $X_1$ and $X_2$ be two universes and let $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X_1\}$, $B = \{< y, \mu_B(y),\ \nu_B(y) > | y \in X_2\}$ be two IFSs; A-over $X_1$ and B-over $X_2$. The Cartesian product $\times$ is defined by $A \times B = \{<< x, y >,\ \mu_A(x)\mu_B(y),\ \nu_A(x)\nu_B(y) > | x \in X_1,\ y \in X_2\}$

**2.4 Definition [2]**

Let α, β ∈ [0, 1].Given IFS A, Atanassov[2] defined the operator

$G_{\alpha,\beta}(A) = \{< x, \alpha\mu_A(x), \beta\nu_A(x) > | x \in X\}$

**2.5 Result**

For every real numbers a and b,

$$\max(a, b) + \min(a, b) = a + b$$

$$\max(a, b) \cdot \min(a, b) = a \cdot b$$

**3. New operators defined on IFSs**

Let A, B ∈ IFS(X). Then we define the following:

i) $A \cup_2 B = \{< x,\ max\left(\frac{\sqrt{\mu_A(x)}}{2}, \frac{\sqrt{\mu_B(x)}}{2}\right), \min\left(\frac{\sqrt{\nu_A(x)}}{2}, \frac{\sqrt{\nu_B(x)}}{2}\right) > |x \in X\}$

ii) $A \cap_2 B = \{< x, min\left(\frac{\sqrt{\mu_A(x)}}{2}, \frac{\sqrt{\mu_B(x)}}{2}\right), \max\left(\frac{\sqrt{\nu_A(x)}}{2}, \frac{\sqrt{\nu_B(x)}}{2}\right) > |x \in X\}$

iii) $A +_2 B = \{< x,\ \frac{\sqrt{\mu_A(x)}}{2} + \frac{\sqrt{\mu_B(x)}}{2} - \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4} > |x \in X\}$

iv) $A \cdot_2 B = \{< x, \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)}}{2} + \frac{\sqrt{\nu_B(x)}}{2} - \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4} > |x \in X\}$

v) $A\ @_2 B = \{< x,\ \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}{4} > |x \in X\}$

vi) $A\ \$_2\ B = \{< x,\ \frac{\sqrt{\mu_A(x)\mu_B(x)}}{2}, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{2} > |x \in X\}$

vii) $A\ \#_2\ B = \{< x,\ \frac{\sqrt{\mu_A(x)\mu_B(x)}}{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}} > |x \in X\}$

viii) $A *_2 B = \{< x, \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{\sqrt{\mu_A(x)\mu_B(x)} + 4}, \frac{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}{\sqrt{\nu_A(x)\nu_B(x)} + 4} > |x \in X\}$

We observe that the above are IFSs. Next we prove some new equalities connected with the above proposed operators.

**Theorem 3.1 .** Let A, B ∈ IFS(X). Then $(A\ \$_2\ B)\ @\ (A\ \$\ B) = G_{\frac{3}{4},\frac{3}{4}}(A\ \$\ B)$

Proof.

From the definitions, .

$$A\ \$_2\ B = \{< x,\ \frac{\sqrt{\mu_A(x)\mu_B(x)}}{2}, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{2} > |x \in X\}$$

$$A\ \$\ B = \{< x,\ \sqrt{\mu_A(x)\mu_B(x)}, \sqrt{\nu_A(x)\nu_B(x)} > |x \in X\}$$

Then

$$(A\ \$_2\ B)\ @\ (A\ \$\ B) = \{< x,\ \frac{\frac{\sqrt{\mu_A(x)\mu_B(x)}}{2} + \sqrt{\mu_A(x)\mu_B(x)}}{2}, \frac{\frac{\sqrt{\nu_A(x)\nu_B(x)}}{2} + \sqrt{\nu_A(x)\nu_B(x)}}{2} > |x \in X\}$$

$$= \{< x,\ \frac{3}{4}\sqrt{\mu_A(x)\mu_B(x)}\ ,\frac{3}{4}\sqrt{\nu_A(x)\nu_B(x)} > |x \in X\}$$

$$= G_{\frac{3}{4},\ \frac{3}{4}}(A\ \$\ B)$$

**Theorem 3.2 .** For every two IFSs A and B we have

i) $(A \cup_2 B) * (A \cap_2 B) = A *_2 B$

ii) $(A *_2 B)\ @\ (A *_2 B) = A\ @_2\ B$

iii) $(A *_2 B)\ \$\ (A *_2 B) = A *_2 B$

Proof.

From the definitions of $A \cup_2 B$ and $A \cap_2 B$ ,

$$(A \cup_2 B) * (A \cap_2 B) = \{< x,\ max\left(\frac{\sqrt{\mu_A(x)}}{2},\ \frac{\sqrt{\mu_B(x)}}{2}\right), \min\left(\frac{\sqrt{\nu_A(x)}}{2},\frac{\sqrt{\nu_B(x)}}{2}\right) > |x \in X\} *$$

$$\{< x,\ min\left(\frac{\sqrt{\mu_A(x)}}{2},\ \frac{\sqrt{\mu_B(x)}}{2}\right), \max\left(\frac{\sqrt{\nu_A(x)}}{2},\ \frac{\sqrt{\nu_B(x)}}{2}\right) > |x \in X\}$$

$$= \{< x,\ \frac{\frac{\sqrt{\mu_A(x)}}{2} + \frac{\sqrt{\mu_B(x)}}{2}}{2\left(\frac{\sqrt{\mu_A(x)\mu_B(x)}}{4} + 1\right)}, \frac{\frac{\sqrt{\nu_A(x)}}{2} + \frac{\sqrt{\nu_B(x)}}{2}}{2\left(\frac{\sqrt{\nu_A(x)\nu_B(x)}}{4} + 1\right)} > |x \in X\}$$

$$= \{< x, \frac{\frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{2}}{2\left(\frac{\sqrt{\mu_A(x)\mu_B(x)} + 4}{4}\right)},\ \frac{\frac{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}{2}}{2\left(\frac{\sqrt{\nu_A(x)\nu_B(x)} + 4}{4}\right)} > |x \in X\}$$

$$= \{< x,\ \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{\sqrt{\mu_A(x)\mu_B(x)} + 4},\ \frac{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}{\sqrt{\nu_A(x)\nu_B(x)} + 4} > |x \in X\}$$

$= A *_2 B$ . The proof of (ii) and (iii) are similar to that of (i).

**Theorem 3.3** Let A, B $\in$ IFS(X). Then

$([(A \cdot B) \cup (A \cdot_2 B)]\ @\ [(A + B) \cup (A +_2 B)])\ @\ ([(A \cdot B) \cap (A \cdot_2 B)]\ @\ [(A + B) \cap (A +_2 B)])$
=

$(A\ @\ B)\ @\ (A\ @_2\ B)$

Proof.

From the definitions,

$$(A \cdot B) \cup (A \cdot_2 B) = \{< x, \max\left(\mu_A(x)\mu_B(x), \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}\right),$$

$$\min\left(\nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x), \frac{\sqrt{\nu_A(x)}}{2} + \frac{\sqrt{\nu_B(x)}}{2} - \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4}\right) > |x \in X\}$$

(3.1)

$$= \{< x, \mu_A(x)\mu_B(x), \frac{\sqrt{\nu_A(x)}}{2} + \frac{\sqrt{\nu_B(x)}}{2} - \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4} > |x \in X\}$$

Similarly

$$(A + B) \cup (A +_2 B) = \{< x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4} > |x \in X\}$$

(3.2)

Apply @ with (3.1) and (3.2) we have

$$[(A \cdot B) \cup (A \cdot_2 B)] \,@\, [(A + B) \cup (A +_2 B)] = \{< x, \frac{\mu_A(x) + \mu_B(x)}{2}, \frac{\sqrt{\nu_A(x) + \nu_B(x)}}{4} > |x \in X\}$$

(3.3)

From the definitions,

$$(A \cdot B) \cap (A \cdot_2 B) = \{< x, \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}, \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x) > |x \in X\}$$

(3.4)

$$(A + B) \cap (A +_2 B) = \{< x, \frac{\sqrt{\mu_A(x)}}{2} + \frac{\sqrt{\mu_B(x)}}{2} - \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}, \nu_A(x)\nu_B(x) > |x \in X\}$$

(3.5)

Apply @ with (3.4) and (3.5) we have

$$[(A \cdot B) \cap (A \cdot_2 B)] \,@\, [(A + B) \cap (A +_2 B)] = \{< x, \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{4}, \frac{\nu_A(x) + \nu_B(x)}{2} > |x \in X\}$$

(3.6)

Apply @ with (3.3) and (3.6) we have

$$([(A \cdot B) \cup (A \cdot_2 B)] \,@\, [(A + B) \cup (A +_2 B)]) \,@\, ([(A \cdot B) \cap (A \cdot_2 B)] \,@\, [(A + B) \cap (A +_2 B)])$$

$$= \{< x, \frac{\mu_A(x) + \mu_B(x)}{4} + \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{8}, \frac{\nu_A(x) + \nu_B(x)}{4} + \frac{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}{8} > |x \in X\}$$

$= \{< x, \frac{\mu_A(x) + \mu_B(x)}{2}, \frac{\nu_A(x) + \nu_B(x)}{2} > | x \in X\} \ @ \ \{< x, \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{8}, \frac{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}{8}$

$> | x \in X\}$

= (A @ B) @ ( A $@_2$ B)

**Theorem 3.4** Let A , B ∈ IFS(X). Then

$$[(A +_2 B) \cup (A \#_2 B)] \ @ \ [(A \#_2 B)] \cap (A \cdot_2 B)] = A \ @_2 B$$

Proof.

From the definitions of $A +_2 B$, $A \cdot_2 B$ and $A \#_2 B$, we have

$$(A +_2 B) \cup (A \#_2 B) = \{< x, \max\left(\frac{\sqrt{\mu_A(x)}}{2} + \frac{\sqrt{\mu_B(x)}}{2} - \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}, \frac{\sqrt{\mu_A(x)\mu_B(x)}}{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}\right),$$

$$\min\left(\frac{\sqrt{\nu_A(x)}}{2} + \frac{\sqrt{\nu_B(x)}}{2} - \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}\right) > | x \in$$

$X\}$

$$= \{< x, \frac{\sqrt{\mu_A(x)}}{2} + \frac{\sqrt{\mu_B(x)}}{2} - \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}} > | x \in X\}$$

(3.7)

$$\text{and } (A \#_2 B) \cap (A \cdot_2 B) = \{< x, \min\left(\frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}, \frac{\sqrt{\mu_A(x)\mu_B(x)}}{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}\right),$$

$$\max\left(\frac{\sqrt{\nu_A(x)\nu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)}}{2} + \frac{\sqrt{\nu_B(x)}}{2} - \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4} > | x \in X\}\right)$$

$$= \{< x, \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)}}{2} + \frac{\sqrt{\nu_B(x)}}{2} - \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4} > | x \in X\}$$

(3.8)

From (3.7) and (3.8)

$[(A +_2 B) \cup (A \#_2 B)] \ @ \ [(A \#_2 B)] \cap (A \cdot_2 B)]$

$$= \{< x, \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}{4} > | x \in X\}$$

$= A \ @_2 B$

**Theorem 3.5** Let A, B ∈ IFS(X). Then

$[(A +_2 B) \cap (A \ @_2 B)] \times [(A +_2 B) \cap (A \#_2 B)] = G_{\frac{1}{2},\frac{1}{2}}(A \ \$_2 \ B) = G_{\frac{1}{4},\frac{1}{4}}(A \ \$ \ B)$

Proof.

From the definitions,

$$(A +_2 B) \cap (A @_2 B) = \{<< x, x >, \min\left(\frac{\sqrt{\mu_A(x)}}{2} + \frac{\sqrt{\mu_B(x)}}{2} - \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}, \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{4}\right),$$

$$\max\left(\frac{\sqrt{\nu_A(x)\nu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}{4}\right) > | x \in X\}$$

$$= \{<< x, x >, \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}{4} > | x \in X\}$$

Similarly

$$(A +_2 B) \cap (A \#_2 B) = \{<< x, x >, \frac{\sqrt{\mu_A(x)\mu_B(x)}}{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}} > | x \in X\}$$

Hence

$[(A +_2 B) \cap (A @_2 B)] \times [(A +_2 B) \cap (A \#_2 B)]$

$$= \{<< x, x >, \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4} > | x \in X\}$$

$$= G_{\frac{1}{4},\frac{1}{4}}(A \$ B) \quad \text{(or)} \quad G_{\frac{1}{2},\frac{1}{2}}(A \$_2 B)$$

**Theorem 3.6** For A, B ∈ IFS(X),

i) $[\,((A \cup_2 B) \cdot (A \cap_2 B)) @ (A +_2 B)] \times_1 (A \#_2 B) = G_{\frac{1}{2},\frac{1}{2}}(A \$_2 B) = G_{\frac{1}{4},\frac{1}{4}}(A \$ B)$

ii) $[(A \cup_2 B) \# (A \cap_2 B)] \times_1 [(A \cup_2 B) @ (A \cap_2 B)] = G_{\frac{1}{2},\frac{1}{2}}(A \$_2 B)$

iii) $[((A \cup_2 B) \cdot (A \cap_2 B)) @ (A +_2 B)] \times_1 [(A \cup_2 B) \# (A \cap_2 B)] = G_{\frac{1}{2},\frac{1}{2}}(A \$_2 B)$

Proof.

By applying the definitions of $[(A \cup_2 B)$, $(A \cap_2 B)$, $(A +_2 B)$ and $(A \#_2 B)$,

$$(A \cup_2 B) \cdot (A \cap_2 B) = \{<< x, x >, \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}, \frac{\sqrt{\mu_A(x)}}{2} + \frac{\sqrt{\mu_B(x)}}{2} - \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4} > | x \in X\}$$

and

$$[((A \cup_2 B) \cdot (A \cap_2 B)) @ (A +_2 B)] = \{<< x, x >, \frac{\frac{\sqrt{\mu_A(x)}}{2} + \frac{\sqrt{\mu_B(x)}}{2} - \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4} + \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}}{2},$$

$$\frac{\frac{\sqrt{\nu_A(x)}}{2} + \frac{\sqrt{\nu_B(x)}}{2} - \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4} + \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4}}{2} > | x \in X\}$$

$$= \{<< x, x >, \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}{4} > | x \in X\}$$

Hence $[(((A \cup_2 B) \cdot (A \cap_2 B)) @ (A +_2 B)] \times (A \#_2 B)$

$$= \{<< x, x >, \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{4} \frac{\sqrt{\mu_A(x)\mu_B(x)}}{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}, \frac{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}{4} \frac{\sqrt{\nu_A(x)\nu_B(x)}}{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}$$

$> | x \in X\}$

$$= \{<< x, x >, \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4} > | x \in X\}$$

$$= G_{\frac{1}{4},\frac{1}{4}}(A \$ B) = G_{\frac{1}{2},\frac{1}{2}}(A \$_2 B)$$

Proof of (ii) and (iii) are similar to that of (i).

**Theorem 3.7** For every A, B ∈ IFS(X),

$$[(A^c \to B) +_2 (A \to B^c)^c] @ [(A^c \to B) \cdot_2 (A \to B^c)^c] = A @_2 B$$

Proof

From the definitions

$A^c \to B = \{ < x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) > | x \in X\}$ and

$(A \to B^c)^c = \{< x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) > | x \in X\}$

Then from the definition of '$+_2$'

$$(A^c \to B) +_2 (A \to B^c)^c = \{< x, \frac{\sqrt{\max(\mu_A(x), \mu_B(x))}}{2} + \frac{\sqrt{\min(\mu_A(x), \mu_B(x))}}{2} - \frac{\sqrt{\max(\mu_A(x), \mu_B(x))}\sqrt{\min(\mu_A(x), \mu_B(x))}}{4} \frac{\sqrt{\min(\nu_A(x), \nu_B(x))}}{2} \frac{\sqrt{\max(\nu_A(x), \nu_B(x))}}{2} > | x \in X\}$$

$$= \{< x, \max\left(\frac{\sqrt{\mu_A(x)}}{2}, \frac{\sqrt{\mu_B(x)}}{2}\right) + \min\left(\frac{\sqrt{\mu_A(x)}}{2}, \frac{\sqrt{\mu_B(x)}}{2}\right) - \frac{\max\left(\sqrt{\mu_A(x)}, \sqrt{\mu_B(x)}\right) \min\left(\sqrt{\mu_A(x)}, \sqrt{\mu_B(x)}\right)}{4}, \min\left(\frac{\sqrt{\nu_A(x)}}{2}, \frac{\sqrt{\nu_B(x)}}{2}\right) \max\left(\frac{\sqrt{\nu_A(x)}}{2}, \frac{\sqrt{\nu_B(x)}}{2}\right) > | x \in X\}$$

$$= \{< x, \frac{\sqrt{\mu_A(x)}}{2} + \frac{\sqrt{\mu_B(x)}}{2} - \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4} > | x \in X\}$$

Similarly

$$(A^c \to B) \cdot_2 (A \to B^c)^c = \{< x, \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)}}{2} + \frac{\sqrt{\nu_B(x)}}{2} - \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4} > | x \in X\}$$

We have $[(A^c \to B) +_2 (A \to B^c)^c] @ [(A^c \to B) \cdot_2 (A \to B^c)^c]$

$$= \{< x, \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}{4} > | x \in X\}$$

$$= A @_2 B$$

**Theorem 3.8** Let A, B ∈ IFS (X). Then

$[(A @_2 B) \to (A \$_2 B)^c] @ [(A @_2 B)^c \to (A \$_2 B)]^c = [(A \$_2 B) @ (A @_2 B)]^c$

Proof

From the definitions,

$$A @_2 B = \{< x, \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}{4} > | x \in X\} \text{ and}$$

$$(A \$ B)^c = \{< x, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{2} \frac{\sqrt{\mu_A(x)\mu_B(x)}}{2} > | x \in X\}$$

Then from the definition of '→'

$$(A @_2 B) \to (A \$ B)^c = \{< x, \max\left(\frac{\sqrt{\nu_A(x)\nu_B(x)}}{2}, \frac{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}{4}\right),$$

$$\min\left(\frac{\sqrt{\mu_A(x)\mu_B(x)}}{2}, \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{4}\right) > | x \in X\}$$

$$= \{< x, \frac{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}{4}, \frac{\sqrt{\mu_A(x)\mu_B(x)}}{2} > | x \in X\}$$

Similarly

$$(A @_2 B)^c \to (A \$ B) = \{< x, \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{2} > | x \in X\} \text{ and}$$

$$[(A @_2 B)^c \to (A \$ B)]^c = \{< x, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{2}, \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{4} > | x \in X\}$$

We have

$[(A @_2 B) \to (A \$ B)^c] @ [(A @_2 B)^c \to (A \$ B)]^c$

$$= \{< x, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4} + \frac{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}{8}, \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{8} + \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4} > | x \in X\}$$

$= [(A \$_2 B) @ (A @_2 B)]^c$

**Theorem 3.9** Let A, B ∈ IFS (X). Then

$[(A \cdot_2 B)^c \to (A +_2 B)] @ [(A \cdot_2 B) \to (A +_2 B)^c]^c = A @_2 B$

Proof

From the definitions,

$$(A \cdot_2 B) \to (A +_2 B)^c = \{< x, \max\left(\frac{\sqrt{\nu_A(x)}}{2} + \frac{\sqrt{\nu_B(x)}}{2} - \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{2}\right),$$

$$\min\left(\frac{\sqrt{\mu_A(x)\mu_B(x)}}{2}, \frac{\sqrt{\mu_A(x)}}{2} + \frac{\sqrt{\mu_B(x)}}{2} - \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}\right) > |x \in X\}$$

$$= \{< x, \frac{\sqrt{\nu_A(x)}}{2} + \frac{\sqrt{\nu_B(x)}}{2} - \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4}, \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4} > |x \in X\}$$

Similarly

$$[(A \cdot_2 B)^c \to (A +_2 B)] = \{< x, \frac{\sqrt{\mu_A(x)}}{2} + \frac{\sqrt{\mu_B(x)}}{2} - \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4} > |x \in X\}$$

and

$$[(A \cdot_2 B)^c \to (A +_2 B)]^c = \{< x, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{4}, \frac{\sqrt{\mu_A(x)}}{2} + \frac{\sqrt{\mu_B(x)}}{2} - \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4} > |x \in X\}$$

We have

$$[(A \cdot_2 B)^c \to (A +_2 B)] @ [(A \cdot_2 B) \to (A +_2 B)^c]^c$$

$$= \{< x, \frac{\sqrt{\mu_A(x)} + \sqrt{\mu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)} + \sqrt{\nu_B(x)}}{4} > |x \in X\}$$

$$= A @_2 B$$

**Theorem 3.10** For every A, B ∈ IFS (X).

$$[(A +_2 B) \to (A @_2 B)^c] \to [(A \cdot_2 B)^c \to (A @_2 B)]^c = A @_2 B$$

Proof

From the definitions,

$$(A +_2 B) \to (A @_2 B)^c = \{< x, \max\left(\frac{\sqrt{\nu_A(x)\nu_B(x)}}{4}, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{2}\right),$$

$$\min\left(\frac{\sqrt{\mu_A(x)}}{2} + \frac{\sqrt{\mu_B(x)}}{2} - \frac{\sqrt{\mu_A(x)\mu_B(x)}}{4}, \frac{\sqrt{\mu_A(x)\mu_B(x)}}{2}\right) > |x \in X\}$$

$$= \{< x, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{2}, \frac{\sqrt{\mu_A(x)\mu_B(x)}}{2} > |x \in X\}$$

Similarly

$$(A \cdot_2 B)^c \rightarrow \left(A @_2 B\right) = \{< x, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{2}, \frac{\sqrt{\mu_A(x)\mu_B(x)}}{2} > | x \in X\} \text{ and}$$

$$[(A \cdot_2 B)^c \rightarrow \left(A @_2 B\right)]^c = \{< x, \frac{\sqrt{\mu_A(x)\mu_B(x)}}{2}, \frac{\sqrt{\nu_A(x)\nu_B(x)}}{2} > | x \in X\}$$

We have

$$[ (A +_2 B) \rightarrow \left(A @_2 B\right)^c] \rightarrow \left[(A \cdot_2 B)^c \rightarrow \left(A @_2 B\right)\right]^c = A @_2 B$$

## Concluding Remark

We extend the set theoretic and arithmetic operations defined on intuitionistic fuzzy sets by R.K.Verma and B.D.Sharma[9,10]. Some results based on the proposed operators are proved. The theorems proved here provide deep study of operators on IFSs. From this study there is a scope of further development on these IFS operators.

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