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On Axially Symmetric Fields in GeneralRelativity

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Abstract

Some of the known axially symmetric fields have been derived through the study of the principal invariants of the Ricci tensor and the curvature tensor by employing Einstein-Rosen time dependent metric.

Keywords: Ricci tensor; Axially symmetric.

1 Introduction

The most important characteristic of axially symmetric fields is their vital role in the study of gravitational radiation which provides a crucial link between general relativity and the macroscopic frontier of physics. Einstein and Rosen (1937) have obtained time dependent cylindrically symmetric metric,

$$ds^2 = e^{2A-2B}(dt^2 - dR^2) - R^2 e^{-2B} d\varphi^2 - e^{2B} dz^2, \quad (1)$$

where A and B are functions of R and t. This metric predicts cylindrical gravitational waves. The importance of the above Einstein-Rosen (ER) metric is well known in the literature. We, in this paper. have considered ER metric and ob- tained some known solutions through the considerations of principal invariants of the Ricci tensor K_{ij} and curvature tensor K_{hijk}.

The principal invariants of these tensors are given by

$$det\left(K^i_j - p\delta^i_j\right) = 0, \quad (2)$$

$$det\left(\overline{K}^\alpha_\beta - q\overline{\delta}^\alpha_\beta\right) = 0 \quad (3)$$

where $\overline{K}_{\alpha\beta} = K_{hijk}$ and the indices α, β run from 1 to 6. The other symbols havethe conventional meaning of Takeno (1966).

Obviously, from the equations (2) and (3), we can write the principal invariants as

$$(p) = (p_1, p_2, p_3, p_4) \tag{4}$$

$$(q) = (q_1, q_2, q_3, q_4, q_5, q_6) \tag{5}$$

2 Some Static Solutions

Case I:

$$Supposing\ \ p_2 = p_3 = p_4, \tag{6}$$

from eq. (2), we get

$$B'' + \frac{B'}{R} = 0 \tag{7}$$

and

$$2\left(B'' + \frac{B'}{R}\right) - \left(A'' + \frac{A'}{R}\right) = 0, \tag{8}$$

where $A' = \frac{dA}{dR}$ etc.

The above equations (7) and (8) are satisfied by

$$A = C_1\ logR + Const.$$

and

$$B = C_2\ logR + Const.,$$

with C_1 and C_2 as the arbitrary constants of integration. Adjusting the constants we then obtain

$$ds^2 = \left(\frac{b_1}{b_2}\right)^2 R^{2(C_1 - C_2)}(dt^2 - dR^2) - \left[\frac{R^{(-c_2)}}{b_2}\right]^2 d\phi^2 - (b_2 R^{c_2})^2\, dz^2, \tag{9}$$

where b_1 and b_2 are arbitrary constants.

We thus find that the choice of $(p) = (p_1, p_2, p_2, p_2)$ yields the space-time (9). It is interesting to note that, in this case, $p_2 = 0$.

Adjusting the constants C_1 and C_2 in such a way that $C_1 = C_2^2$, we get $(p) = (0, 0, 0, 0)$ and consequently the well known Levi-Civita metric

$$ds^2 = \left(\frac{R}{R_0}\right)^{\left(\frac{m^2+2m}{2}\right)}(dt^2 - dR^2) - R^2\left(\frac{R}{R_0}\right)^m d\phi^2 - \left(\frac{R}{R_0}\right)^{-m} dz^2, \qquad (10)$$

is obtained.

Furthermore, if the principal invariants of curvature tensor are chosen in such a way that

$$(q) = (q_1, q_1, q_3, q_4, q_4, q_3),$$

We obtain the Levi-Civita metric (10)

$$q_1 = -P,\ q_3 = -\left(\frac{-mP}{2}\right),\ q_1 + q_3 + q_4 = 0,$$

where

$$P = \left(\frac{R}{R_0}\right)^{\left(-\frac{m^2+2m}{2}\right)} \cdot \frac{(m^2+2m)}{4R^2}$$

Case II:

Choosing $q^{'}s$ such that

$$q_1 = 0 \ and \ q_2 = 0$$

we get

$$A = log(C_1 R) + C_2 R$$

and

$$B = log(C_3 R)$$

where $C^{'}s$ are arbitrary constants.

The space-time (1) then, absorbing the constants in the differentials, assumes theform

$$ds^2 = e^{2C_2R}(dt^2 - dR^2) - d\varphi^2 - R^2 dz^2. \qquad (11)$$

Case III:

With the suppposition

$$q_2 = 0, q_5 + q_6 = 0,$$

one obtains

$$A = log(C_1 R)$$

and

$$B = log(C_2 R).$$

The space-time (1) can now be written as

$$ds^2 = (dt^2 - dR^2) - d\varphi^2 - R^2 dz^2 \qquad (12)$$

3 Some Time Dependent Solution

Case (I):

If we postulate

$$A = A(R) \; and \; B = B(t),$$

we obtain

$$(p) = (p_1, 0, 0, p_4).$$

Furthermore, taking the choice $p_1 = 0$ and $p_4 = 0$, we get

$$ds^2 = e^{\left(\frac{m^2 R^2}{4}\right) + mt}(dt^2 - dR^2) - R^2 e^{mt} d\phi^2 - e^{-mt} dz^2, \qquad (13)$$

which is the metric obtained by Misra and Radhakrishna (1962).

If $(p) = (p_1, 0, 0, 0)$, we find

$$A'' + \frac{A'}{R} = 2\dot{B}^2, \quad \dot{B} = \frac{dB}{dt}. \qquad (14)$$

Equation (14) yields

$$A = C_1 \, logR + \frac{(C_2 R)^2}{2} + C_3,$$

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$$B = C_2 t + C_4$$

and then the metric (1) can easily be constructed from the values of A and B.

Case (II):

Assuming

$$(p) = (p_1, p_2, p_2, p_4),$$

We get

$$B'' + \frac{B'}{R} - \ddot{B} = 0. \tag{15}$$

This equation (15) has its own importance from physics point of view. It corresponds to the cylindrical and gravitational waves. The various aspects of (15)

are discussed by Rosen (1954). and Karade (1978).

The other two $p's$ i.e. (p_1, p_4) are given by the quadratic equation

$$p^2 - p\left(K^1_1 + K^4_4\right) + K^1_1 K^4_4 - K^1_4 + K^4_1 = 0. \tag{16}$$

3.3 Case (iii):

For the general case the six $q's$, determined by the equation (3), are given by

$$q_1 = K^{14}_{14}, \; q_2 = K^{23}_{23}$$

and

$$\left[\left(K^{12}_{12} - q\right)\left(K^{24}_{24} - q\right) - K^{24}_{12} K^{12}_{24}\right]$$

$$\left[\left(K^{13}_{13} - q\right)\left(K^{34}_{34} - q\right) - K^{13}_{34} K^{34}_{13}\right] = 0. \tag{17}$$

Restricting $q's$ such that

$$K^{12}_{12} = K^{13}_{13} \; and \; K^{24}_{24} = K^{34}_{34},$$

one gets the equation (15) corresponding to the cylindrical gravitational wave.

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