International Journal of Engineering, Science and Mathematics
Vol. 8 Issue 9, September 2019,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

ESTIMATION AND PREDICTION OF POPULATION GROWTH IN INDIA BY MATHEMATICAL MODELS

Shakar Hussain.
Deparment of Mathematics,
Rabindranath Tagore University, Mendua,
Post: Bhojpur, Near Bangrasiya Chouraha,Bhopal-Chiklod Road,
Madhya Pradesh – 464993.

Abstract: *In this research paper, we used mathematical models to foresee the population growth in India. India is the second most populated nation after China on the earth's planet with about a fifth of the total population [Wikipedia].To expect the population growth in India from 1971 to 2055 by two mathematical models i.e., exponential and logistic. The information utilized was gathered from population pyramid.Net. It was examined sing MATLAB programming and it precisely fitted the logistic growth curve. The Exponential model predicts a development pace of 2.3% per annum and furthermore predicts the population to be 3826783879 in 2055. Population growth of any country relies upon the vital coefficients [from Malthus model]. We decided together the carrying capacity and the vital coefficients 'a' and 'b' are 0.02 and* $3.0325769575 \times 10^{-11}$. *As per the Logistic model, the population distribution is 2% and target population is 2990164790 in 2055. We calculated the Mean Absolute Percentage Error (MAPE) for exponential and logistic model is 5.48% and 2.75% definitely.*

KEYWORDS: Population growth, MAPE, Vital coefficients, Exponential growth model and Logistic growth model.

INTRODUCTION:

Population size and growth of a country eventually affects the economy, strategy, science and natural environment of the country and decide investigating the expense of normal resources. Projection of any nation's population assumes a noteworthy job in the organizing manner just as in the basic leadership for commercial and statistic improvement. As we know the difficult matter of the world is the colossal growth of population particularly in the developing nations like India. Each legislature and aggregate parts consistently require accurate prediction regarding the future population growth. The statisticians and mathematicians analyze the previous information and results of population for projection of the variables. There is colossal worry about the outcome of human population growth for social and commercial development. Heightening every one of these issues is in the population growth. Mathematical modeling is a method which includes numerical techniques. That is it is a systematic way of emulating reality by utilizing the language of arithmetic, time or in connection to one another. A model can have different structure. It is critical to underline that a model isn't actual world however only a human develop to enable us to better

see true configuration. One uses model in all perspective to our life, so as to remove the difficulties in projection of population growth. Everyone expect that the projection from the mathematical models archaic with the real world data. Here we applied the population models to foresee the population change with time. This research is related with the projection of India's population growth by utilizing two models i.e., exponential and logistic definitely.

MATERIALS AND METHODS:

Research is a right way to reach at archaic solution for the issues through the methodical gathering, investigation and translation of information. In connection to this research, optional arranged yearly population information of India from 1971-2019 (complete) were gathered from population pyramid.net. The two models i.e., exponential as well as logistic model were utilized to process the anticipated population growth and draw the table of real and anticipated population values on time in year, utilizing MATLAB. The Integrity of attack of the model is surveyed utilizing the 'Mean Absolute Percentage Error' (MAPE).

EXPONENTIAL GROWTH MODEL:

A scientific model of population growth put forward by R. Malthus in 1798. He suggested by the presumption that the population develops at a rate relative to the size of the population. This is a sensible presumption for a population of a microorganisms or creature under perfect conditions (boundless condition, satisfactory sustenance, nonappearance of vulture, and resistance from illness).

Assuming the population is P_0^G at time t = t_0 and we are keen on projecting the population P^G at time t =t_1. As it were we need to discover a function which is called a population function $P^G(t)$ for $t_0 \leq t \leq t_1$ satisfying $P^G(t_0) = P_0{}^G$.

Now the initial value problem,

$$\frac{dP^G}{dt} = kP^G(t), t_0 \leq t_1; P^G(t_0) = P_0{}^G$$

(1.1)

Integrating the above equation by variable separable;

$$\int \frac{dP^G}{P^G} = K\int dt$$

ln P^G = Kt + C, where C is constant of integration.

$$P^G(t) = P_0{}^G e^{Kt}$$

Or

$$P^G(t) = P_0{}^G e^{K(t-t_0)}$$

(1.2) where K is called Malthus factor and multiplication of this constant determines the growth rate. Eq.(1.1) is a differential equation because it have an unknown function P^G and its derivative $\frac{dP^G}{dt}$ and Eq.(1.2) is the solution of differential eq.(1.1) in the exponential form. By this Model, we predicted the population growth .If exclude a population of 0, then $P^G(t) > 0$ for all t. This implies the population is continually expanding. Actually, as $P^G(t)$ increases, equation (1.1) demonstrates that $\frac{dP^G}{dt}$ increases. As such if population increases the growth rate also increases. Eq. (1.1) is proper for displaying population growth in perfect circumstances, in this way we need to perceive that an increasingly practical must mirror the reality a given condition has a bounded resources.

LOGISTIC GROWTH MODEL:

The Belgian Mathematician Verhulst in 1840's put forward a logistic growth model. In this model, he uses the concept of upper limit i.e., carrying capacity. Therefore this model does not shows only the size of population but also shows how much this size is deviated from the carrying capacity (most supportable population).Verhulst modifies the Malthus model and suggest that population size is directly proportional to past as well as current population.

$$\frac{a-bP^G(t)}{a}$$

(1.3) Vital coefficients of the population are ‘a’ and ‘b’. The above equation delineates the deviation of population from its upper limit i.e., carrying capacity. Presently the value of population pick up nearer to $\frac{a}{b}$, the value in eq.(1.3) becomes very small and tends to 0. It helps us to bind the population growth. Therefore the changed equation using this new concept is:

$$\frac{dP^G}{dt}=\frac{aP^G(t)\left(a-bP^G(t)\right)}{a}, t_0 \le t \le t_1; P^G(t_0)=P_0{}^G$$

(1.4)

Eq. (1.4) is called as Logistic Population growth model. After solving (1.4) then applied the initial conditions, so the equation (1.4) becomes

$$\frac{dP^G}{dt}=aP^G-b\left(P^G\right)^2$$

(1.5)

Now applying the Separation of variables in eq. (1.5) and integrate it, we get

$$\int\frac{1}{a}\left(\frac{1}{P^G}+\frac{b}{a-bp^G}\right)dP^G=t+C, \text{ so that}$$

$$\frac{1}{a}\left(\ln P^G-\ln\left(a-bp^G\right)\right)=t+C$$

(1.6)

At time, t = 0 and $P^G = P_0^G$, we see that C = $\frac{1}{a}\left(\ln P_0^G-\ln\left(a-bP_0^G\right)\right)$.Equation (1.6) becomes

$$\frac{1}{a}\left(\ln P^G-\ln\left(a-bp^G\right)\right)=t+\frac{1}{a}\left(\ln P_0^G-\ln\left(a-bP_0^G\right)\right)$$

To solve the above term, we obtain,

$$P^G=\frac{\frac{a}{b}}{1+\left(\frac{\frac{a}{b}}{P_0^G}-1\right)e^{-at}}$$

(1.7)

Now taking the limits as t approaches to ∞, we have

$$P_{max}{}^G=\lim_{t\to\infty}P^G=\frac{a}{b},(a>0)$$

(1.8)

Twice differentiating equation (1.7) with respect to t, we get

$$\frac{d^2P^G}{dt^2} = \frac{C^*a^3e^{at}\left(C^* - e^{at}\right)}{b\left(C^* + e^{at}\right)^3}$$

(1.9)

Where $C^* = \frac{\frac{a}{b}}{P_0^G} - 1$

At the point of inflection the second derivative of $P^G = 0$. It will possible only if

$$C^* = e^{at}$$

(1.10)

Now solve the eq. (1.10) w.r.t t, then it becomes

$$t = \frac{\ln C^*}{a}$$

(1.11)

At this time the point of inflection occurs i.e., the value of population is half of its carrying capacity. When the point of inflection takes place then let time, t=t_k

Suppose if time, t = 1 and t = 2, the values of P^G are P_1^G and P_2^G, after that we will get from equation (1.7) is

$$\frac{b}{a}\left(1 - e^{-a}\right) = \frac{1}{P_1^G} - \frac{e^{-a}}{P_0^G}$$

(1.12)

$$\frac{b}{a}\left(1 - e^{-2a}\right) = \frac{1}{P_2^G} - \frac{e^{-2a}}{P_0^G}$$

(1.13)

Now dividing the equation (1.10) by (1.9), we get

$$1 + e^{-a} = \frac{\frac{1}{P_2^G} - \frac{e^{-2a}}{P_0^G}}{\frac{1}{P_1^G} - \frac{e^{-a}}{P_0^G}}$$

In this way $e^{-a} = \frac{P_0^G\left(P_2^G - P_1^G\right)}{P_2^G\left(P_1^G - P_0^G\right)}$

(1.14)

Putting e^{-a} in eq. (1.9), we obtain

$$\frac{b}{a} = \frac{\left(P_1^G\right)^2 - P_0^G P_2^G}{P_1^G\left(P_0^G P_1^G - 2P_0^G P_2^G + P_1^G P_2^G\right)}$$

(1.15)

Thus the bounded value of P^G is given by

$$P^G{}_{\max} = \lim_{t\to\infty} P^G = \frac{a}{b} = \frac{P_1^G\left(P_0^G P_1^G - 2P_0^G P_2^G + P_1^G P_2^g\right)}{\left(P_1^G\right)^2 - P_0^G P_2^G}$$

(1.16)

MEAN ABSOLUTE PERCENTAGE ERROR (MAPE):

It is a statistical instrument which is used to describe the %age error and accuracy of the unique model of the population estimations. Its concept is very important in statistical calculation and error. Its mathematical equation is written as

MAPE = $\frac{1}{n}\sum_{t}^{n}\left|\frac{A_t-\bar{F}_t}{A_t}\right|\times 100$ (1.17)

Where $A_t, \bar{F}_t$ and n are real, fitted forecast and number of calculation related with population.

If the errors are small then the MAPE values are also small. The following values are given by Lewis in 1982 if MAPE value < 10% is highly accurate projection, if MAPE value is between10% - 20% shows good projection and if MAPE values is 21% -50% is sensible projection and above 51% we assume that projection is inaccurate.

RESULTS AND DISCUSSION:

To predict India's incoming population from the actual population data of India by Exponential population growth model which is described in eq. (1.2). The actual population data of India has written on table 1. From the table when t = 0 against the year 1971 then P_0^G =566605402 .Now we have to find out the 'K' which is growth rate, for this we assume P_1^G = 579800632 at t = 1.

$$579800632 = 566605402 \times e^k$$

$$k = \ln\left(\frac{579800632}{566605402}\right)$$

$$k = 0.02302118$$

Thus the General Solution is

$$P^G(t) = 566605402 \times e^{0.02302118} \quad (1.18)$$

This tells us that the population prediction growth rate of India is 2.3% by exponential growth model. With this procedure we can projected the India's population upto 2055.

From table 1, put t = 0, 1 & 2 against the years 1971, 1972 & 1973 respectively.

So, P_0^G = 566605402, P_1^G = 579800632 and P_2^G = 593451889. Putting these values of P_0^G, P_1^G and P_2^G in (1.16) we obtain $P_{\max} = \frac{a}{b} = 659505110$. This is the predicted India's population carrying capacity.

We get from equation (1.14), $e^{-a} \approx 0.98$ therefore $a \approx -\ln(0.98)$. Thus the value of 'a' is $"a" \approx 0.02$. It tells us that the predicted India's population growth rate is 2% by the logistic growth model.

By using $\frac{a}{b} = 659505110$ and eq. (1.16) so it gives b =3.0325769575 $\times 10^{-11}$.

Putting the value of P_0^G, e^{-a} and $\frac{a}{b}$ in equation (1.7), we obtain

$$P^G(t) = \frac{659505110}{1+(0.1639583873929)\times(0.02)^t} \quad (1.19)$$

This general solution will be used to forecast India's population upto 2055. The values of India's population projection by two models have shown below on table 1.

Table 1: India's population projection by Exponential and Logistic Growth Model

Year	Actual population	Projected Population	
		Exponential Model	Logistic Model
1971	566605402	566605402	566605402
1972	579800632	579637326	577937510
1973	593451889	592968985	589496260
1974	607446519	606607272	601286185
1975	621703641	620559239	613311908
1976	636182810	634832102	625578146
1977	650907559	649433240	638089708
1978	665936435	664370205	650851502
1979	681358553	679650720	663868532
1980	697229745	695282687	677145902
1981	713561406	711274189	690688820
1982	730303461	727633495	704502596
1983	747374856	744369065	718592647
1984	764664278	761489554	732964499
1985	782685127	779003814	747623788
1986	799607235	796920902	762576263
1987	817232241	815250083	777827788
1988	834944397	834000835	793384343
1989	852736160	853182854	809252029
1990	870601776	872806060	825437069
1991	888513869	892880599	841945810
1992	906461358	913416853	858784726
1993	924475633	934425441	875960420
1994	942604211	955917226	893479628
1995	960874982	977903322	911349220

1996	979290432	1000395098	929576204
1997	997817250	1023404185	948167728
1998	1016402907	1046942478	967131082
1999	1034976626	1071022154	986473703
2000	1053481072	1095655663	1006203177
2001	1071888190	1120855743	1026327240
2002	1090189358	1146635425	1046853784
2003	1108369577	1173008039	1067790859
2004	1126419321	1199987223	1089146676
2005	1144326293	1227586929	1110929609
2006	1162088305	1255821428	1133148201
2007	1179685631	1284705320	1155811165
2008	1197070109	1314253542	1178927388
2009	1214182182	1344481373	1202505935
2010	1230984504	1375404444	1226556053
2011	1247446011	1407038746	1251087174
2012	1263589639	1439400637	1276108917
2013	1279498874	1472506851	1301631095
2014	1295291543	1506374508	1327663716
2015	1311050527	1541021121	1354216990
2016	1326801576	1576464606	1381301329
2017	1342512706	1612723291	1408927355
2018	1358137719	1649815926	1437105902
2019	1373605068	1687761692	1465848020
2020		1726580210	1495164980
2021		1766291554	1525068279
2022		1806916259	1555569644
2023		1848475332	1586681036
2024		1890990264	1618414656
2025		1934483040	1650782949
2026		1978976149	1683798607

2027		2024492600	1717474579
2028		2071055929	1751824070
2029		2118690215	1786860551
2030		2167420089	1822597762
2031		2217270751	1859049712
2032		2268267978	1896230706
2033		2320438141	1934155320
2034		2373808218	1972838426
2035		2428405807	2012295194
2036		2484259140	2052541097
2037		2541397100	2093591918
2038		2599849233	2135463756
2039		2659645765	2178173031
2040		2720817617	2221736491
2041		2783396422	2266171220
2042		2847414539	2311494644
2043		2912905073	2357724536
2044		2979901889	2404879026
2045		3048439632	2452976606
2046		3118553743	2502036138
2047		3190280479	2552076860
2048		3263656930	2603118397
2049		3338721039	2655180764
2050		3415511622	2708284379
2051		3494068389	2762450066
2052		3574431961	2817699067
2053		3656643896	2874053048
2054		3740746705	2931534108
2055		3826783879	2990164790
Mean Absolute Percentage Error		**5.48%**	**2.75%**

Source: Population pyramid.Net

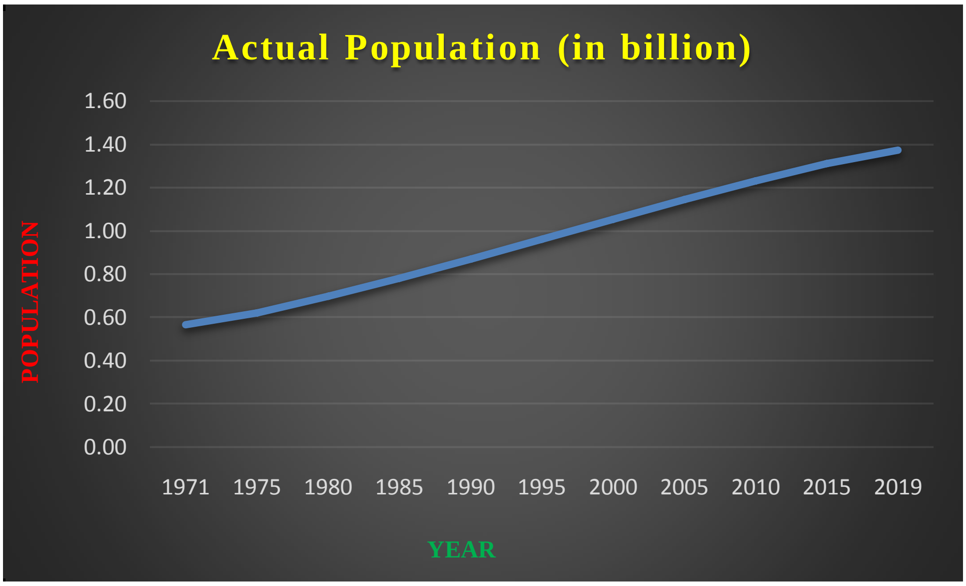

Figure 1(a): Actual population graph from 1971 - 2019.

Fig. 1(a) delineates that from 1971 the population of India has expanded all through. This might be help in the following fields like education, agribusiness, water supply and wellbeing administrations. The exponential model anticipated India's population to be 3826783879 though the Logistic model anticipated it to be 2990164790.This is exhibited on Figure 1(b). We found that the Mean Absolute Percentage Error (MAPE) by using eq. (1.17) of the two models that is exponential and logistic model are 5.48% and 2.75% definitely.

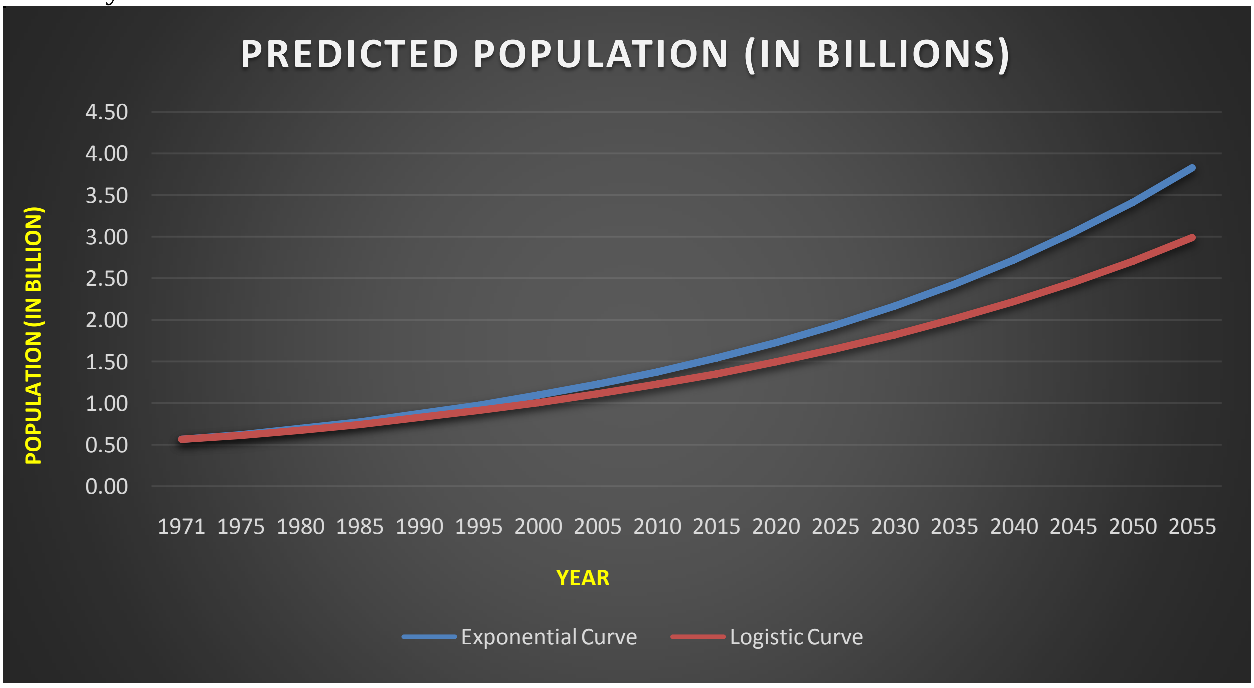

Figure 1(b): Predicted population graph corresponds to time.

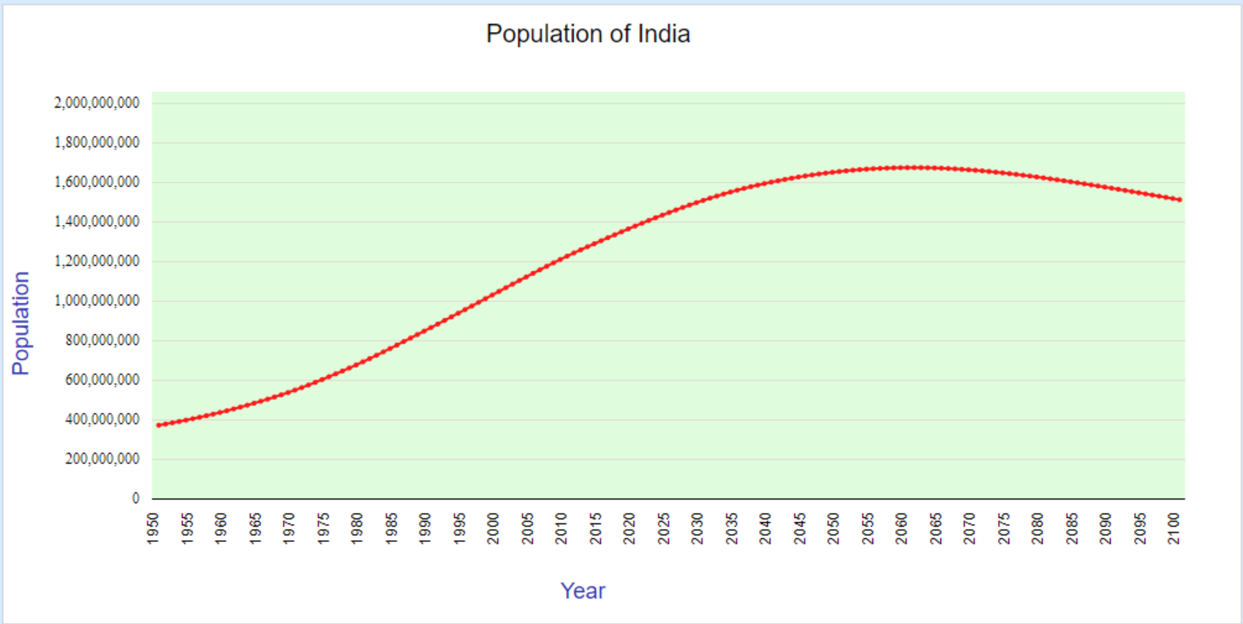

Figure1(c): Predicted population values from 1950 to 2100 (picture taken from Statistics Times.com).

Figure 1(b) demonstrates the diagram of the anticipated population of India with the two models and Figure1(c), shows that anticipated population esteems is a S-type curve. Now by comparing fig. 1(b) and fig.1(c) we determined that our models that is exponential and logistic models fitted well into the anticipation population curve [fig.1(c)] which shows that population growth rate begins at an exponential phase until it gets closer to the carrying capacity and after that its growth rate getting slow down and pass to a stable one.

CONCLUSION:

We determined that the anticipated India's population by Exponential Model in the year 2055 will be 3826783879 at 2.3% growth rate with a MAPE of 5.48%. The logistic Model then again anticipated a carrying capacity with respect to the number of inhabitants in India to be 2990164790. Population growth of any nation relies upon the vital coefficients so the vital coefficients 'a' & 'b' are 0.02 & $3.0325769575 \times 10^{-11}$ separately. Therefore India's population growth, as indicated by this model, is 2% per annum. It likewise anticipated the population of India to be 2990164790 in 2055 with a MAPE of 2.75%. Based on Lewis (1982) statement we can say that the logistic growth model have a better projection result than the exponential growth model. As we know from the above study that the vital coefficients are very important because number of things like technology, environmental pollution and some social changes depends upon these vital coefficient. Hence they should be calculated after every two or three years to check the variation in the growth rate of population.

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