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TREES

BAIDYANATH YADAV (Teacher) & **Dr. K.K. Choudhary (Retd.)**
J.B. N. S. H/S Madneshwar sthan, Babubarhi, M.R.M. College, Darbhanga

Abstract :- Acyclic Graph

An acyclic graph is one that contains no cycles. A tree is a connected acyclic graph. The tree on six Vertices are shown in figure-

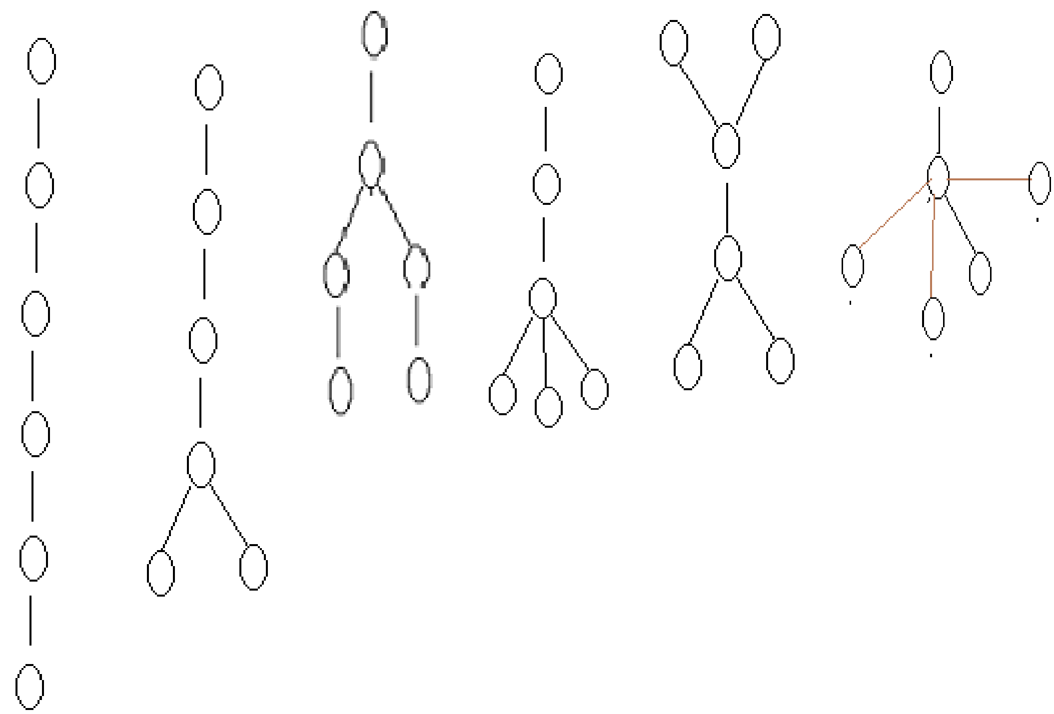

Figure :

Theorem : "In a tree, any two vertices are connected by a unique path.

Proof : By contradiction, let G be a tree and assume that there are two distinct (u,v.)- Path P_1, and P_2 in G. Since $P_1 \neq P_2$, there is an edge $e = xy$ of P_1 that is not an edge of P_2. Clearly the graph $(P_1 \cup P_2)$-e is connected. It therefore contains an (x, y)-path P. But then P+e is a cycle in the acyclic graph G, a contradiction.

The converse of this theorem hold for graphs without loope.

Observe that all the trees on six vertices have five edges. In general :

Theorem : If G is a tree, then $\varepsilon = v - 1$

Proof :- By induction on v. When v=1, G≅K and $\varepsilon = 0 = v-1$.

Suppose the theorem true for all trees on fewer than v vertices, and let G be a tree on $v \geq 2$ vertices. Let $uv \in E$. Then G-uv contains no (u, v). Path, since uv is the unique (u, v) - path in G.

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Thus G-uv` is disconnected and so ω(G-uv)=2. The components G_1 and G_2 of G-uv being acyclic, are trees. Moreover, each has fewer than v vertices. Therefore, by the induction hypothesis

$\varepsilon(G_i)=v(G_i)-1$ for $i=1., 2, ...$

Thus, $\varepsilon(G)=\varepsilon(G_1)+\varepsilon(G_2)+1=v(G_1)+V(G_2)-1=V(G)-1$

Corollary :- Every nontrivial tree has at least two vertices of degree one.

Proof, Let G be a nontrivial tree, Then d(v)≥1, for all v∈V

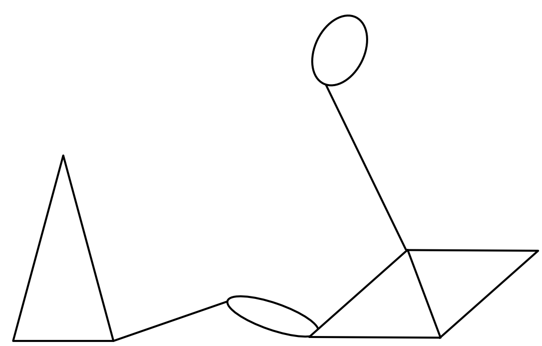
Also, consider the incidence matrix M. The sum of the entries in the row corresponding to vertex v is precisely d(v), and therefore $\sum_{v\in V} d(v)$ is just the sum of all entries in M.

But this sum is also 2ε, since each of the ε column sums of M is 2.

i.e. $\sum_{v\in V} d(v) = 2\varepsilon = 2v-2$

It now follows that d(v)=1 for at least two vertices v.

Cut Vertices : A vertex v of G is a cut vertex if E can be partioned into two nonempty subsets E_1 and E_2 such that $G[E_1]$ and $G[E_2]$ have just the vertex v in common. If G is loopless and nontrivial, then v is a cut vertex of G if and only if ω(G-v)>ω(G). The graph of figure has the five cut vertices indicated.



Theorem : A vertex v of a tree G is a cut vertex of G if and only if d(v)>1.

Proof : If d(v)=0, G≅K, and clearly, v is not a cut vertex.

If d(v)=1, G - v is an acyclic graph with v(G-v) - 1 edges. Hence ω(G-v)=1 = ω(G), and v is not a cut vertex of G.

If d(v)>1, there are distinct vertices u and w adjacent to v. The path uvw is a (u,w)-path in G. Therefore we know uvw is the unique (u,w)-path in G. It follows that there is no (u,w)-path in G-v, and therefore that ω(G-v)>1 = ω(G). Thus v is a cut vertex of G.

Corollary : Every nontrivial loopless connected graph has at least two vertices that are not cut vertices.

Proof : Let G be a nontrivial loopless connected graph. Every connected graph, G contains a spanning tree T. and every nontrivial tree has at least two vertices of degree one and A vertex v of a tree G is a cut vertex of G if and only if dv>1, T has at least two vertices that are not cut vertices. Let v be any such vertex. Then,

ω(T-v) = 1

Since T is a spanning subgraph of G, T-v is a spanning subgraph of G-v and therefore

ω(G-v) ≤ ω (T-v)

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If follows that ω(G-v) = 1, and hence that v is not a cut vertex of G. Since there are at least two such vertices v, the proof is complete.

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