
FUZZY TIME SERIES FORECASTING IN ENROLLMENT BY INTERVAL METHOD

V.VANITHA , Lecturer in Mathematics, Tamil Nadu Polytechnic College, Madurai.
Email: vanithaveluchamy80@gmail.com

Abstract

Forecasting results are mostly affected by length of the intervals in Fuzzy time series forecasting. In this paper, the rule for determining the length of interval is proposed. Based on the new intervals, fuzzy logical relationship is derived and more accuracy results are got than the existing methods in the historical enrollments of the University of Alabama and the results are compared.

Keywords: Fuzzy Time Series, Fuzzy Sets, Fuzzy logical relationships, Fuzzy forecasting,
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I. Introduction.

The definitions and properties of fuzzy time series forecasting are found in Zadeh(1975), Wu(1986) and Song and Chissom(1993,1994). Sulliwán and Woodal(1994) described a Markov model using linguistic values directly but with membership function of the fuzzy approach replaced by analogous probability function. In the place of complicated maximum – minimum composition operators Chen (1996) used a simple arithmetic operation for time series forecasting. Hwang , Chen and Lee(1998) presented a method of forecasting enrollments using fuzzy time series based on the concept that the variation of enrollment of this year is related to the trend of the enrollments of the past years. Song and Chissom (1993,1994) proposed time variant and time invariant methods whose observation are linguistic values. They asserted that all traditional forecasting methods not suitable when the historical enrollment data are composed of linguistic values, Forecasting using fuzzy times series has been widely used in many activities. Huarng(2001) indicated that the length of intervals will affect the forecasting accuracy rate and a proper choice of length of the intervals can enhance the forecasting result. He presented the distribution length approach and the average based length approach to deal with forecasting probabilities based on the intervals with different lengths. Chen(2002) presented a method of forecasting based on high-order fuzzy time series. Chen and Hsu(2004) proposed a first order time variant method for enrollment forecasting using fuzzy time series. Also M. Sah and Y.D Konstantin(2005) presented a first order fuzzy time series method of forecasting. Chen et.al(2009) presented clustering techniques for clustering historical enrollments in to intervals of different lengths. Lee.et.al(2009) proposed the modified weighted method for enrollment forecasting. Wang and Chen(2009) presented a forecasting method based on clustering techniques and two factors higher order fuzzy time series.

In order to get a higher forecasting accuracy rate, in this paper, a new attractive simple method is presented to forecast the enrollments of the University of Alabama based in interval length, weightage factor and fuzzy logical relationships

Fuzzy Time Series

Some concepts from Song and Chissom (1993,1994) are reviewed. The difference between the fuzzy time series and conventional time series that the value of the former are fuzzy sets while the values of the later are real numbers.

Define the Universe of Discourse U , find the minimum enrollment D_{\min} and the maximum enrollment D_{\max} from the known historical data. Based on the D_{\min} and D_{\max} , define U as $[D_{\min} - d_1$

, $D_{\max} + d_2$] where d_1 and d_2 are the two positive numbers. Partition the universe of discourse in to intervals u_1, u_2, \dots, u_n .

A fuzzy set of U is defined by $A = \frac{\mu_A(u_1)}{u_1} + \frac{\mu_A(u_2)}{u_2} + \dots + \frac{\mu_A(u_n)}{u_n}$ where μ_A is the membership function of A , $\mu_A: U \rightarrow [0,1]$ and $\mu_A(u_i)$ indicates the grade of membership of u_i in A , where $\mu_A(u_i) \in [0,1]$ and $1 \leq i \leq n$.

Definition: Let $Y(t)$ ($t = \dots, 0, 1, 2, \dots$), a subset of \mathbf{R} , be the universe of discourse on which fuzzy sets $f_i(t)$ ($i = 1, 2, 3, \dots$) are defined and let $F(t)$ be the collection of $f_i(t)$ ($i = 1, 2, 3, \dots$). Then $F(t)$ is called a fuzzy time series on $Y(t)$ ($t = \dots, 0, 1, 2, \dots$)

We see from the definition that $F(t)$ can be regarded as a linguistic variable and $f_i(t)$ ($i = 1, 2, 3, \dots$) can be viewed as possible linguistic values of $F(t)$, where $f_i(t)$ ($i = 1, 2, 3, \dots$) are represented by fuzzy sets. If $F(t)$ is caused by $F(t-1)$ only, then this relationship is represented by $F(t-1) \rightarrow F(t)$. If $F(t-1) = A_i$ and $F(t) = A_j$ where A_i and A_j are fuzzy sets, then the fuzzy logical relationship between $F(t-1)$ and $F(t)$ can be represented by $A_i \rightarrow A_j$ where A_i and A_j are called the current state and the next state of the fuzzy logical relationship respectively.

Length of Interval

For enrollment forecasting, Song and Chissom (1993, 1994) choose 1000 as the length of intervals without specifying any reason. Since then 1000 has been used as the length of intervals in further studies. In fact, different length of intervals may lead to different forecasting results. Kunhuang Huarng (2001) presented a method in which the length of the intervals is 400. Chen et al (2009) presented a method for forecasting the enrollments of the University of Alabama based on automatic clustering techniques and fuzzy logical relationships. The automatic clustering techniques generate 21 intervals with different lengths of intervals. Vamitha et al (2015) presented the technique of dividing the intervals into 18 unequal lengths of intervals. However, the Mean Square Error (M.S.E) is very less compared to other methods when 21 intervals are divided into 105 intervals with different lengths. This is the motivation to propose a simple method to generate intervals with even lengths.

Step 1: Sorting the actual enrollment in an ascending sequence.

Step 2: Obtain the difference between the adjacent values.

Step 3: Calculate the range and the frequency.

Step 4: Compute the length of interval as

$$\text{Length of interval} = \frac{\text{Max. Difference} - \text{Min. Difference}}{n}$$

Where n is the highest frequency of the difference value.

Proposed Method

In this section, a new method for forecasting enrollments based on the proposed length of interval, weightage factor and fuzzy logical relationships is presented.

Step 1: Apply the proposed length of interval to obtain the intervals and calculate the midpoints of each interval. Add the calculated length with the first value of the sorted sequence gives the first interval. With the end point of the first interval add the calculated length gives the second interval. Proceed up to the last value of the sorted sequence.

Step 2:

Assume that there are n intervals u_2, \dots, u_n . Then using trapezoidal membership rule define each fuzzy set A_i , $1 \leq i \leq n$ as follows:

$$A_1 = \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \dots + \frac{0}{u_{n-1}} + \frac{0}{u_n}$$

$$A_2 = \frac{0.5}{u_1} + \frac{1}{u_2} + \frac{0.5}{u_3} + \frac{0}{u_4} + \dots + \frac{0}{u_{n-1}} + \frac{0}{u_n}$$

$$A_3 = \frac{0}{u_1} + \frac{0.5}{u_2} + \frac{1}{u_3} + \frac{0.5}{u_4} + \dots + \frac{0}{u_{n-1}} + \frac{0}{u_n}$$

⋮

⋮

⋮

$$A_n = \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \dots + \frac{0.5}{u_{n-1}} + \frac{1}{u_n}$$

Step3:

Fuzzify each data in the historical enrollments in to a fuzzy set. If the data belongs to u_i , where $1 \leq i \leq n$, then the data is fuzzified in to A_i .

Step 4: Construct the fuzzy logical relationship (FLR) based on the fuzzified historical enrollments obtained in step3 . If the fuzzified enrollments of year's t and $t+1$ are A_i and A_j respectively, then construct the fuzzy logical relationships $A_i \rightarrow A_j$ where A_i and A_j are the current state and the next state of fuzzy logical relationships respectively. Divide the fuzzy logical relationships in to fuzzy logical relationship group(FLG), where the fuzzy logical relationships having the same current state are put into the same fuzzy logical relationship group.

Step5:

Rule 1: If $A_i \rightarrow A_j$ is a FLG , then the forecasting value corresponding A_j is m_j , where m_j is the midvalue of the interval u_j .

Rule 2 : Let $A_i \rightarrow A_j^{(a)}$, $A_k^{(b)}$, $A_l^{(1)}$, ... be a FLG where A_j and A_k are repeated for a and b times respectively etc, then the forecasting value of A_j is average of m_j, m_k, m_l .

In the following , we apply the proposed method to forecast the enrollment of the University of Alabama shown in Table !. The length of interval , the intervals and the midpoints of the intervals are as follows:

$D_{min} = 13055, D_{max} = 19337.$

$D_1 = 55, D_2 = 663$ and Define $U = [13000, 20000]$

Actual enrollment in an ascending sequence:

13055,13563,13867,14696,15145,15163,15311,15433,15460,15497,15603,
15861,15984,16388,16807,16859,16919,18150,18876,18970,19328, 19337.

Differences:

508,304,829,449,18,148,102,27,37,106,258,123,404,19,52,60,**1231**,726,94, 358,**9**.

We calculate the range and frequency of the difference by following method

Range - Frequency

0-10 - 1

10-100 - 7

100-1000 - 12

1000-10000 -1

Here $n = 12$.

When we perform the above steps we can get

Length of intervals = $1231-9/12 = 102$

No.of intervals = $19337-13055/102 = 62$.

the length of interval as **62** and also we get the following intervals:

$u_1 = [13055, 13157],$

$u_2 = [13157, 13259], \dots u_{62} = [19277, 19379]$

After calculating the midpoint of each interval u_i where $1 \leq i \leq 108$, we get the results as follows.

$$m_1 = 13106$$

$$m_2 = 13208, \dots, m_{62} = 19328.$$

Table 1: Fuzzified and Forecasted enrollments of the University of Alabama.

Year	Actual Enrollment (A_v)	Fuzzified Enrollment	Forecasted Enrollment (F_v)
1971	13055	A_1	-
1972	13563	A_5	13514
1973	13867	A_8	13820
1974	14696	A_{17}	14738
1975	15460	A_{24}	15452
1976	15311	A_{23}	15316
1977	15603	A_{25}	15554
1978	15861	A_{28}	15860
1979	16807	A_{37}	16778
1980	16919	A_{38}	16880
1981	16388	A_{33}	17237
1982	15433	A_{24}	15452
1983	15497	A_{24}	15316
1984	15145	A_{21}	15316
1985	15163	A_{21}	15146
1986	15984	A_{29}	15962
1987	16859	A_{38}	16880
1988	18150	A_{50}	17237
1989	18970	A_{58}	18920
1990	19328	A_{62}	19328
1991	19337	A_{62}	19328
1992	18876	A_{58}	18920

Then the definitions of the fuzzy sets A_1, A_2, \dots, A_{108} are as follows:

$$A_1 = \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \dots + \frac{0}{u_{n-1}} + \frac{0}{u_n}$$

$$A_2 = \frac{0.5}{u_1} + \frac{1}{u_2} + \frac{0.5}{u_3} + \frac{0}{u_4} + \dots + \frac{0}{u_{n-1}} + \frac{0}{u_n}$$

$$A_3 = \frac{0}{u_1} + \frac{0.5}{u_2} + \frac{1}{u_3} + \frac{0.5}{u_4} + \dots + \frac{0}{u_{n-1}} + \frac{0}{u_n}$$

⋮
⋮
⋮

$$A_{108} = \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \dots + \frac{0.5}{u_{n-1}} + \frac{1}{u_n}$$

Fuzzy logical relationships:

$$A_1 \rightarrow A_5 : \quad A_5 \rightarrow A_8 : \quad A_8 \rightarrow A_{17} : \quad A_{17} \rightarrow A_{24} : \quad A_{24} \rightarrow A_{23} :$$

$$A_{23} \rightarrow A_{25} : \quad A_{25} \rightarrow A_{28} : \quad A_{28} \rightarrow A_{37} : \quad A_{37} \rightarrow A_{38} : \quad A_{38} \rightarrow A_{33} :$$

$$A_{33} \rightarrow A_{24} : \quad A_{24} \rightarrow A_{24} : \quad A_{24} \rightarrow A_{21} : \quad A_{21} \rightarrow A_{21} : \quad A_{21} \rightarrow A_{29} : \\ A_{29} \rightarrow A_{38} : \quad A_{38} \rightarrow A_{50} : \quad A_{50} \rightarrow A_{58} : \quad A_{58} \rightarrow A_{62} : \quad A_{62} \rightarrow A_{62} :$$

$$A_{62} \rightarrow A_{58} :$$

Fuzzy logical relationship group:

$$A_1 \rightarrow A_5 : \quad A_5 \rightarrow A_8 : \quad A_8 \rightarrow A_{17} : \\ A_{17} \rightarrow A_{24} : \quad A_{21} \rightarrow A_{21}, A_{29} : \quad A_{23} \rightarrow A_{25} : \\ A_{24} \rightarrow A_{23}, A_{24}, A_{21} : \quad A_{25} \rightarrow A_{28} : \quad A_{28} \rightarrow A_{37} : \\ A_{38} \rightarrow A_{50} : \quad A_{50} \rightarrow A_{58} : \quad A_{58} \rightarrow A_{62} : \\ A_{62} \rightarrow A_{62} : \quad A_{62} \rightarrow A_{62}, A_{58} :$$

Experimental Results

To evaluate the forecasting performance, four fuzzy time series methods are adopted for comparing of their forecasting results with those obtained by proposed method. The mean absolute percentage error (MAPE) is used to evaluate the forecasting result accuracy.

The formula is:

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{A_{vi} - F_{vi}}{A_{vi}} \right| \times 100$$

Index	Chen(1996)	Hwang et.al(1998)	Lee and Chou (2004)	Proposed method
MAPE	3.08	2.94	2.69	1.02

Conclusion

In this paper, we have developed a new fuzzy time series forecasting method for forecasting enrollment of the University of Alabama based on interval length, weightage factor and fuzzy logical relationships. In other words the proposed method gets a higher accuracy rate than Chen(1996), Hwang et.al(1998), Lee(2004).The rate of accuracy corresponds to minimizing the length of intervals and maximizing the number of intervals.

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