International Journal of Management, IT & Engineering
Vol. 9 Issue 1, January 2019,
ISSN: 2249-0558 Impact Factor: 7.119
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Eco Friendly Fully Fuzzy Multi objective Multi-item Solid Transportation problem

P.Vijayalakshmi*

Dr.J.Merline vinotha**

Abstract

Transportation sector generates the largest share of greenhouse gas emissions. In real life situation due to uncontrollable factor the parameters of the transportation problem may not known precisely. In some practical situation the decision maker is interested in setting multi aspiration levels for objectives that may not be expressed in a specific manner. The transportation related emissions are the major source of air pollutants and another source of affecting environment is emissions from packaging .So this paper focuses on Eco friendly Fully Fuzzy multi objective Multi-item solid transportation problem (FFMOMISTP). Two models have been discussed in which one consisting of emissions related to diesel and Non eco-friendly packaging and another one is emissions related to biodiesel and eco-friendly packaging. Using This proposed methodology ecofriendly FFMOMISTP is converted into 4 crisp problems and the optimal solution is obtained by using goal programming approach through LINGO software.The main advantage of proposed method is the optimal solutions are also in non-negative LR flat fuzzy numbers. Finally the Aspiration levels of the above two models have been compared.

Key words: Solid transportation problem, LR flat fuzzy numbers, Emissions related to fuels and packaging, goal programming approach.

Introduction

Transportation problem is a particular class of linear programming , which is associated with day-to-day activities in our real life and mainly deals with logistics. It helps in solving problems on distribution and transportation of resources from one place to another.In many situations the parameters of transportation problem may not be deterministic due to uncontrollable factors like lack of information and uncertainty in judgment the values of the transportation parameters .This impreciseness in the values of the parameters can be represented by the fuzzy numbers. The fuzzy set was introduced by Zadeh[24]. Many authors[2,3,4] have used the fuzzy numbers to represent the uncertainty in transportation parameters and proposed methods to solve them .

In real life application many researchers most often used triangular and trapezoidal shapes of membership functions for representing fuzzy numbers[23,28]. The advantages of using triangular or trapezoidal fuzzy number are easy to use and easy to interpret. In some application the other shapes may be preferable like power membership function and Gaussian membership function. In addition triangular, trapezoidal, power and Gaussian fuzzy numbers are LR flat fuzzy numbers. Due to these features, in this paper all parameters are considered as LR flat fuzzy numbers.

* Assistant professor,Hindusthan Institute of technology,Coimbatore

** Assistant professor,Holy cross college (Autonomous),Trichy

The conventional transportation problem proposed by Hitchcock in 1941 [11] was generalized to the transportation problem in which it takes three types of constraints namely, source, destination and conveyance constraints instead of only source and destination constraints. This generalized problem is called the solid transportation problem. It was first stated by Schell in 1955[10] . It plays a major role in business economies as many business situations like this. Haley gives the solution procedure of solid transportation problem.

There are several objectives are considered and optimized in our real world problems. Such problems are called multi objective transportation problem. In a multi objective transportation problem in which more than one items are transported from different source to different destination using different types of conveyances, the problem reduces to multi objective multi-item solid transportation problem (MOMISTP). The MOMSTP model was given by kundu et al. Recently pramanik et al. have developed a multi objective STP in a fuzzy environment. In MOMISTP all the parameters are considered as fuzzy number. Then the problem is reduced to fully fuzzy multi objective multi-item solid transportation problems.

According to Eva Pongracz (2007) "Packaging-related sources of pollution are also electricity generation (CO_2 , SO_2 ,NO_xemissions) and transportation-related emissions (e.g.,CO_2 , SO_2 ,NO_x , dust, hydro carbons). It is increasingly important to take in to account of the transportation- related emissions". Safety cost is to paid by the customer for safety measure used in packaging. If the price of common food is assumed to be 100.The formula for safety cost is expressed as $S = (Q_p - O_p)/100$,where Q_p is the quality price and O_p is an ordinary food price. Bio packaging is the one of best environmental-friendly packaging to reduce the emission during the transportation. Diesel fuel produces many harmful emissions when it is burned, diesel-fueled vehicles are major sources of harmful pollutants such as ground-level ozone and particulate matter.

Scientific research confirms that Biodiesel exhaust has a less harmful impact on human health than petroleum diesel fuel. Biodiesel emissions showed decreased levels of polycyclic aromatic hydrocarbons(PAH) and nitrite PAH compounds which have been identified as potential cancer-causing compounds.

So this paper focuseson eco Friendly FFMOMISTP. The main advantage of proposed method is the optimal solutions are also expressed in non-negative LR flat fuzzy numbers. Also this model will be highly useful to the society as it has been focused to reduce the environment impact of transportation. Tothe best of our knowledge no researchers have used LR flat fuzzy numbers to study MOMISTP .

In Eco Friendly FFMOMISTP all of the parameters such as transportation cost, packaging cost, emission cost, supplies and demands conveyance capacities are considered as LR flat fuzzy numbers .Two models have been discussedin which one consisting of emissions related to diesel and Non eco-friendly packaging and another one is emissions related to biodiesel and eco-friendly packaging. This FFMOMISTP can be converted into four crisp transportation problems and the optimal solution is obtained by using goal programming approach through LINGO software.

Rest of the paper is organized as follows:

In section 2, some basic definitions related to the fuzzy set theory are presented. In section 3, the FFMOMISTPs can be formulated in terms of non-negative LR flat fuzzy numbers. In section 4 , solution methodology is discussed. Numerical examples are solved in section 5 and conclusion is presented in section 6.

2. Preliminaries:

Definition:2.1According to A.Ebrahimnejad (2016) "

A fuzzy number is a convex normalized fuzzy set of the real line $\mathbb{R}$, whose membership function is piecewise continues. We denote the set of fuzzy numbers on $\mathbb{R}$ with F ($\mathbb{R}$)".

Definition: 2.2 According to A.Ebrahimnejad (2016) "

A function L: $[0,\infty) \to [0,1]$ (or R: $[0,\infty) \to [0,1]$) is said to be reference function of fuzzy numbers if and only if i) L (0) =1 (R (0) =1)ii)L (or R) is non-increasing on $[0,\infty)$".

Remark:2.1According to A.Ebrahimnejad (2016) "

The commonly used linear reference functions and non-linear reference functions with parameter q, denoted as RF_q are summarized as follows:

i) Linear max {0,1-X} (ii)Power : $RF_q=$ max $\{0,1-x^q\}$, q>0

iii)Exponential power: $RF_q= e^{-x^q}$,q > 0(iv)Rational : $RF_q= \frac{1}{1+x^q}, q>o$”

Definition:2.3According to A.Ebrahimnejad (2016) “

An LR flat fuzzy number (see Fig 1), is denoted as $\tilde{a}$ = $(a_1, a_2, a_3, a_4)_{LR}$, if the membership function $\mu_{\tilde{a}}(x)$ is given by

$$\mu_{\tilde{a}}(x)=\begin{cases} L\left(\frac{a_2-x}{a_2-a_1}\right), & a_1\le x\le a_2,\\ 1, & a_2\le x\le a_3,\\ R\left(\frac{x-a_3}{a_4-a_3}\right), & a_3\le x\le a_4.\end{cases}$$

The set of LR flat fuzzy numbers on real line is denoted by $\ell\Re(\mathbb{R})$ ”

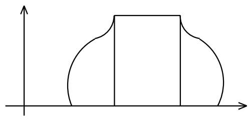

Fig:1. An LR flat fuzzy number $\tilde{a}=(a_1,a_2,a_3a_4)_{LR}$

Remark:2.2According to A.Ebrahimnejad (2016) “

If L(x) = R(x) = max{0,1-x} then the LR fuzzy number $\tilde{a} = (a_1,a_2,a_3,a_4)_{LR}$ is denoted by $\tilde{a}$ = (a_1, a_2, a_3, a_4) and is called a trapezoidal fuzzy number with the following membership function (see Fig. 2):

$$\mu_{\tilde{a}}(x)=\begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1\le x\le a_2\\ 1 & a_2\le x\le a_3\\ \frac{a_4-x}{a_4-a_3}, & a_3\le x\le a_4\end{cases}$$

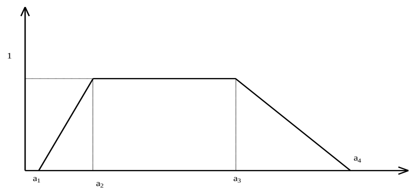

Fig:2. A trapezoidal fuzzy number $\tilde{a}=(a_1,a_2,a_3a_4)$

If $a_2 = a_3$ then the trapezoidal fuzzy number $\tilde{a}$ = (a_1, a_2, a_3, a_4) is called a triangular fuzzy number.”

Remark:2.3According to A.Ebrahimnejad (2016) “

If L (x) = R(x) = e^{-x} then the LR fuzzy number $\tilde{a}$ = $(a_1, a_2, a_3, a_4)_{LR}$ is called an exponential fuzzy number”.

Remark:2.4According to A.Ebrahimnejad (2016) “

If L (x) = R(x) $= e^{-x/2}$ then the LR fuzzy number $\tilde{a}$ = $(a_1, a_2, a_3, a_4)_{LR}$ is called a Gaussian fuzzy number”.

Remark:2.5According to A.Ebrahimnejad (2016) “

If L (x) = R(x) = max $\{0,1-x^2\}$ then the LR Fuzzy number $\tilde{a}$ = $(a_1, a_2, a_3, a_4)_{LR}$ is called a quadratic fuzzy number” .

Definition:2.4According to A.Ebrahimnejad (2016) “

Two LR flat fuzzy numbers $\tilde{a}$ = $(a_1, a_2, a_3, a_4)_{LR}$ and $\tilde{b}$ = $(b_1, b_2, b_3,b_4)_{LR}$ is said to be equal (i.e.) $\tilde{a} = \tilde{b}$ if and only if $a_1=b_1$, $a_2=b_2$, $a_3=b_3$ and $a_4=b_4$

Definition:2.5According to A.Ebrahimnejad (2016) “

An LR flat fuzzy number $\tilde{a}$ = $(a_1, a_2, a_3, a_4)_{LR}$ is said to be a non-negative LR flat fuzzy number if and only if $a_i \ge 0$. The set of all non-negative LR flat fuzzy numbers is denoted by $\ell\Re(\mathbb{R})^+$”.

Definition:2.6According to A.Ebrahimnejad (2016) "

The FFMOMISTP is said to be balanced, if $\tilde{\sum_{i=1,2...m}} \tilde{s}_i = \tilde{\sum_{j=1,2...n}} \tilde{d}_j$, otherwise, it is called unbalanced FFMOMISTP".

Remark : 2.6 According to A.Ebrahimnejad (2016) "

It is assumed that the all parameters of the FFMOMISTP are as non-negative LR flat fuzzy number because the negative quantity of the product and all are represented by non-negative LR flat fuzzy number. Negative transportation costs have no physical meaning".

Definition:2.7According to A.Ebrahimnejad (2016) "

Let $\tilde{a} = (a_1, a_2, a_3, a_4)_{LR}$ and $\tilde{b} = (b_1, b_2, b_3, b_4)_{LR}$ be two LR flat fuzzy numbers. Then, $\tilde{a} \tilde{\le} \tilde{b}$ if and only if $a_1 \le b_1$, $a_2 \le b_2$, $a_3 \le b_3$, $a_4 \le b_4$".

2.8 Arithmetic operations: According to A.Ebrahimnejad (2016) "

Let $\tilde{a} = (a_1, a_2, a_3, a_4)_{LR}$ and $\tilde{b} = (b_1, b_2, b_3, b_4)_{LR}$ be two non-negative LR flat fuzzy numbers and k be a non-negative real number. Then the exact formula for the extended addition, the approximate formula for the extended multiplication and the scalar multiplication are defined as follows [22];

(i) $(a_1, a_2, a_3, a_4)_{LR} \tilde{+} (b_1, b_2, b_3, b_4)_{LR} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)_{LR}$

(ii) $(a_1, a_2, a_3, a_4)_{LR} \otimes (b_1, b_2, b_3, b_4)_{LR} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)_{LR}$

(iii) $K(a_1, a_2, a_3, a_4)_{LR} = (ka_1, ka_2, ka_3, ka_4)_{LR}$"

3. Mathematical formulation of FFMOMISTP:

In real life situation due to uncontrollable factors the values of transportation cost, packing cost, supply, demand and conveyance capacity may not be known precisely. Fuzzy set theory is highly useful to solve such types of transportation problem.

The fuzzy transportation problem, in which a decision maker is uncertain about the precise values of transportation cost, packaging cost, emission cost,supply demand and conveyance capacity may be formulated as follows:

$$\min \tilde{\sum_{p=1,2...l}} \tilde{\sum_{i=1,2...m}} \tilde{\sum_{j=1,2...n}} \tilde{\sum_{k=1,2...o}} \tilde{c}_{ijk}^{tp} \otimes \tilde{x}_{ijk}^{p}, t = 1,2,3.....R$$

$$\text{S.t} \tilde{\sum_{j=1,2,....n}} \tilde{\sum_{k=1,2...o}} \tilde{x}_{ijk}^{p} = \tilde{s}_i^{p}, i = 1,2,...\text{m}, \text{p} = 1,2....\text{l}$$

$$\tilde{\sum_{i=1,2...m}} \tilde{\sum_{k=1,2...o}} \tilde{x}_{ijk}^{p} = \tilde{d}_j^{p}, \quad j = 1,2...n, p = 1,2...\text{l}$$

$$\tilde{\sum_{p=1,2....l}} \tilde{\sum_{i=1,2...m}} \tilde{\sum_{j=1,2...n}} \tilde{x}_{ijk}^{p} = \tilde{e}_k, \quad k = 1,2...o$$

$$\tilde{x}_{ijk}^{p} \in \ell\Re(\,)^{+}, \quad i = 1,2...m, j = 1,2...n, k = 1,2...o, p = 1,2...l$$

Where, t: total number of objectives, m: total number of origins, n: total number of destinations l : total number of items. o: total number of conveyances $\tilde{s}_i$: fuzzy supply of the commodity at i^th origin, $\tilde{d}_j$: The fuzzy demand of the commodity at j^th destination. $\tilde{e}_k$: The fuzzy conveyances capacity, $\tilde{c}_{ijk}$: The fuzzy transportation cost for unit quantity of the commodity from i^th origin to j^th destination with k^th conveyances to minimize the total fuzzy transportation cost $\tilde{x}_{ijk}$: The fuzzy quantity of the commodity from i^th origin to j^th destination with k^th conveyances to minimize the total fuzzy transportation cost.

Here we assume that $\tilde{c}_{ijk}, \tilde{s}_i, \tilde{d}_j, \tilde{e}_k$ and $\tilde{x}_{ijk}$ all are represented by non-negative LR flat fuzzy numbers. $\left(c^p_{ijk,1}, c^p_{ijk,2}, c^p_{ijk,3}, c^p_{ijk,4}\right)_{LR}$, $\left(s^p_{i1}, s^p_{i2}, s^p_{i3}, s^p_{i4}\right)_{LR}$, $\left(d^p_{j1}, d^p_{j2}, d^p_{j3}, d^p_{j4}\right)_{LR}$, $\left(e_{k1}, e_{k2}, e_{k3}, e_{k4}\right)_{LR}$, and $\left(x^p_{ijk,1}, x^p_{ijk,2}, x^p_{ijk,3}, x^p_{ijk,4}\right)_{LR}$, respectively. FFMOMISTP can be reformulated as

$$\min \tilde{\sum_{P=1,2...l}} \tilde{\sum_{i=1,2...m}} \tilde{\sum_{j=1,2,3...n}} \tilde{\sum_{k=1,2...0}} \left(c^{tp}_{ijk,1}, c^{tp}_{ijk,2}, c^{tp}_{ijk,3}, c^{tp}_{ijk,4}\right)_{LR}, \otimes \left(x^p_{ijk,1}, x^p_{ijk,2}, x^p_{ijk,3}, x^p_{ijk,4}\right)_{LR},$$

S.to $\tilde{\sum_{j=1,2...n}} \tilde{\sum_{k=1,2,...0}} \left(x^p_{ijk,1}, x^p_{ijk,2}, x^p_{ijk,3}, x^p_{ijk,4}\right)_{LR} = \left(s^p_{i,1}, s^p_{i,2}, s^p_{i,3}, s^p_{i,4}\right)_{LR} i = 1,2,3...m, p = 1,2...l$ ...2.1

$\tilde{\sum_{i=1,2...m}} \tilde{\sum_{k=1,2...0}} \left(x^p_{ijk,1}, x^p_{ijk,2}, x^p_{ijk,3}, x^p_{ijk,4}\right)_{LR} = \left(d^p_{j,1}, d^p_{j,2}, d^p_{j,3}, d^p_{j,4}\right)_{LR}, j = 1,2,3...n, p = 1,2,3...l$ ...2.2 (2)

$\tilde{\sum_{p=1,2...l}} \tilde{\sum_{i=1,2...m}} \tilde{\sum_{j=1,2...n}} \left(x^p_{ijk,1}, x^p_{ijk,2}, x^p_{ijk,3}, x^p_{ijk,4}\right)_{LR} = \left(e_{k,1}, e_{k,2} e_{k,3}, e_{k,4}\right)_{LR}, k = 1,2.3...o$ ...2.3

$\left(x^p_{ijk,1}, x^p_{ijk,2}, x^p_{ijk,3}, x^p_{ijk,4}\right)_{LR} \in \ell\Re(\,)^+, i = 1,2,3...m, j = 1,2,3...n, k = 1,2,3...o, p = 1,2,3...l$ ...2.4

By definition of 2.4, 2.5 and arithmetic operations 2.8, FFMOMISTP (2) may be rewritten as follows:

$$\min\left[\sum_{P=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{K=1}^{o} c^{tp}_{ijk,1} x^p_{ijk,1}, \sum_{P=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{K=1}^{o} c^{tp}_{ijk,2} x^p_{ijk,2}, \sum_{P=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{K=1}^{o} c^{tp}_{ijk,3} x^p_{ijk,3}, \sum_{P=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{K=1}^{o} c^{tp}_{ijk,4} x^p_{ijk,4}\right]_{LR}$$

S.to $\sum_{j=1}^{n} \sum_{k=1}^{o} x^p_{ijk,1} = s^p_{i,1}\ i = 1,2,...m, p = 1,2...l$ ...3.1

$\sum_{j=1}^{n} \sum_{k=1}^{o} x^p_{ijk,2} = s^p_{i,2}\ i = 1,2,...m, p = 1,2...l$ ...3.2

$\sum_{j=1}^{n} \sum_{k=1}^{o} x^p_{ijk,3} = s^p_{i,3}\ i = 1,2,...m, p = 1,2...l$ ...3.3

$\sum_{j=1}^{n} \sum_{k=1}^{o} x^p_{ijk,4} = s^p_{i,4}\ i = 1,2,...m, p = 1,2...l$ ...3.4

$\sum_{i=1}^{m} \sum_{k=1}^{o} x^p_{ijk,1} = d^p_{j,1}, j = 1,2...n, p = 1,2...l$ ...3.5

$\sum_{i=1}^{m} \sum_{k=1}^{o} x^p_{ijk,2} = d^p_{j,2}, j = 1,2...n, p = 1,2...l$ ...3.6

$\sum_{i=1}^{m} \sum_{k=1}^{o} x^p_{ijk,3} = d^p_{j,3}, j = 1,2...n, p = 1,2...l$ ...3.7

$\sum_{i=1}^{m} \sum_{k=1}^{o} x^p_{ijk,4} = d^p_{j,4}, j = 1,2...n, p = 1,2...l$ ...3.8

$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x^p_{ijk,1} = e_{k,1}\ k = 1,2...o$ ...3.9 (3)

$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x^p_{ijk,2} = e_{k,2}\ k = 1,2...o$ ...3.10

$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x^p_{ijk,3} = e_{k,3}\ k = 1,2...o$ ...3.11

$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x^p_{ijk,4} = e_{k,4}\ k = 1,2...o$ ...3.12

$x^p_{ijk,4} - x^p_{ijk,3} \geq 0, i = 1,2...m, j = 1,2...n, k = 1,2...o, p = 1,2...l$ ...3.13

$x^p_{ijk,3} - x^p_{ijk,2} \geq 0, i = 1,2...m, j = 1,2...n, k = 1,2...o, p = 1,2...l$ ...3.14

$x^{p}_{ijk,2} - x^{p}_{ijk,1} \geq 0, i=1,2...m,\ j=1,2...n, k=1,2...o, p=1,2...l$ ...3.15

$x^{p}_{ijk,1} \geq 0, i=1,2...m,\ j=1,2...n, k=1,2...o, p=1,2...l$ ...3.16

The objective function of model (3) is obtained using the multiplication of two non-negative LR flat fuzzy numbers. Constraints (3.1) to (3.12) in model (3) are simply obtained using the equality of two LR flat fuzzy numbers. To impose the non-negative $\tilde{x}^{p}_{ijk}$ preserving its form as a non-negative LR flat fuzzy number, we have $x^{p}_{ijk,4} \geq x^{p}_{ijk,3} \geq x^{p}_{ijk,2} \geq x^{p}_{ijk,1} \geq 0,$ which corresponds to constraints (3.13) to (3.16).

4. Solution methodology:

The fuzzy transportation problem can be converted into four crisp transportation problem. This is used to reduce the complexity of solving FFMOMISTP (3). In addition the non-negative fuzzy solution can be achieved by using this method.

The constraints (3.13) – (3.15) are used to preserve the form of the optimal solutions as a non-negative LR flat fuzzy numbers. Also it should be noted that Without these constraints, the feasible space of FFMOMISTP (3) is separable in terms of $x^{p}_{ijk,1}, x^{p}_{ijk,2}, x^{p}_{ijk,3},$ and $x^{p}_{ijk,4}$.Due to this fact, this method is proposed for solving FFMOMISTP (3)first by removing the constraints (3.13) – (3.15) from the feasible space and then by decomposing the FFMOMISTP (3) into four crisp transportation problems . It is shown that the integration of optimal solution of four sub-problems not only provides the optimal solution of the FFMOMISTP (3) for each objective .But also it satisfies the constraints (3.13)-(3.15).

4.1. Algorithm:

Step :1 Solve the following crisp transportation problem using LINGO

$$z_1^{*} = \min \sum_{p=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{o} c^{tp}_{ijk,1}\, x^{p}_{ijk,1}$$

$$s.to \sum_{j=1}^{n} \sum_{k=1}^{o} x^{p}_{ijk,1} = s^{p}_{i,1}\ i=1,2,...m, p=1,2...l \quad ...4.1$$

$$\sum_{i=1}^{m} \sum_{k=1}^{o} x^{p}_{ijk,1} = d^{p}_{j,1},\ j=1,2...n, p=1,2...l \quad ...4.2 \qquad (4)$$

$$\sum_{p=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{n} x^{p}_{ijk,1} = e_{k,1}\ \ k=1,2...o \quad ...4.3$$

$$x^{p}_{ijk,1} \geq 0, i=1,2...m,\ j=1,2...n, k=1,2...o, p=1,2...l \quad ...4.4$$

The optimal value of the objective function of model(4), z_1^{*}, computes the first component of fuzzy solution.

Step 2:

Solve the following bounded FFMOMISTP using LINGO with assuming $x_1^{*} = \left(x^{p*}_{ijk,1}\right)_{nm\times 1}$ is the optimal solution of the above problem (4):

$$z_2^{*} = \min \sum_{p=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{o} c^{tp}_{ijk,2}\, x^{p}_{ijk,2}$$

$$s.to \sum_{j=1}^{n} \sum_{k=1}^{o} x^{p}_{ijk,2} = s^{p}_{i,2}\ i=1,2,...m, p=1,2...l \quad ...5.1$$

$$\sum_{i=1}^{m} \sum_{k=1}^{o} x^{p}_{ijk,2} = d^{p}_{j,2},\ j=1,2...n, p=1,2...l \quad ...5.2 \qquad (5)$$

$$\sum_{p=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{n} x^{p}_{ijk,2} = e_{k,2}\ \ k=1,2...o \quad ...5.3$$

$$x^{p}_{ijk,2} \geq x^{p*}_{ijk,1}, i=1,2...m,\ j=1,2....n, k=1,2...o, p=1,2...l \quad ...5.4$$

The optimal value of the objective function of model(5), z_2^*, computes the second component of fuzzy solution.

Proposition: 4.2

The optimal value of the objective function of z_1^* is less than or equal to the optimal value of the objective function of z_2^*.

Proof:

Let $x_2^* = \left(x_{ijk,2}^{p*}\right)_{nm\times 1}$ be the optimal solution of model (5). On the one hand, from the constraints (5.4) it follows that $x_{ijk,1}^{p*} \le x_{ijk,2}^{p*}$. On the other hand due to the form of fuzzy number $\left(c_{ijk,1}^{tp}, c_{ijk,2}^{tp}, c_{ijk,3}^{tp}, c_{ijk,4}^{tp}\right)_{LR}$, We have $c_{ijk,1}^{tp} \le c_{ijk,2}^{tp}$. Therefore $\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,1}^{tp} x_{ijk,1}^{p*} \le \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,2}^{tp} x_{ijk,2}^{p*}$ which means that $z_1^* \le z_2^*$.

Step 3:

Slove the following bounded FFMOMISTP using LINGO with assuming $x_2^* = \left(x_{ijk,2}^{p*}\right)_{nm\times 1}$ is the optimal solution of problem (5).

$$z_3^* = \min \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o} c_{ijk,3}^{tp}\, x_{ijk,3}^{p}$$

$$\text{s.to} \sum_{j=1}^{n}\sum_{k=1}^{o} x_{ijk,3}^{p} = s_{i,3}^{p}\ i = 1,2,\dots m, p = 1,2\dots l \qquad \dots 6.1$$

$$\sum_{i=1}^{m}\sum_{k=1}^{o} x_{ijk,3}^{p} = d_{j,3}^{p},\ j = 1,2\dots n, \mathrm{p} = 1,2\dots l \qquad \dots 6.2 \qquad (6)$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk,3}^{p} = e_{k,3}\ k = 1,2\dots o \qquad \dots 6.3$$

$$x_{ijk,3}^{p} \ge x_{ijk,2}^{p*}, i = 1,2\dots m,\ j = 1,2\dots n, k = 1,2\dots o, p = 1,2\dots l \qquad \dots 6.4$$

The optimal value of the objective function of model (6), z_3^*, computes the third component of fuzzy solution. Moreover, the following statement can be made.

Proposition 4.3: The optimal value of the objective function of z_2^* less than or equal to the optimal value of the objective function of z_3^*.

Step 4: Solve the following bounded FFMOMISTP based upon the LINGO with assuming $x_3^* = \left(x_{ijk,3}^{p*}\right)_{nm\times 1}$ is the optimal solution of (6):

$$z_4^* = \min \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,4}^{tp}\, x_{ijk,4}^{p}$$

$$\text{s.t} \quad \sum_{j=1}^{n}\sum_{k=1}^{o} x_{ijk,4}^{p} = s_{i,4}^{p}\ i = 1,2,\dots m, p = 1,2\dots l \qquad \dots 7.1$$

$$\sum_{i=1}^{m}\sum_{k=1}^{o} x_{ijk,4}^{p} = d_{j,4}^{p},\ j = 1,2\dots n, \mathrm{p} = 1,2\dots l \qquad \dots 7.2 \qquad (7)$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk,4}^{p} = e_{k,4}\ k = 1,2\dots o \qquad \dots 7.3$$

$$x_{ijk,4}^{p} \ge x_{ijk,3}^{p*}, i = 1,2\dots m,\ j = 1,2\dots n, k = 1,2\dots o, p = 1,2\dots l \qquad \dots 7.4.$$

The optimal value of the objective function of model(7), z_4^*, computes the forth component of fuzzy solution. Moreover the following statement can be made.

Proposition4.4:

The optimal value of the objective function of z_3^* is less than or equal to the optimal value of the objective function of z_4^*

Step: 5

Find the fuzzy optimal solution by substituting the values of $x^{p*}_{ijk,1}, x^{p*}_{ijk,2}, x^{p*}_{ijk,3}$ and $x^{p*}_{ijk,4}$ in $\tilde{x}^{p*}_{ijk} = \left(x^{p*}_{ijk,1}, x^{p*}_{ijk,2}, x^{p*}_{ijk,3}, x^{p*}_{ijk,4}\right)_{LR.}$

Step : 6 Find the total minimum fuzzy transportation cost by putting the values of $\tilde{x}^{p*}_{ijk}$ in

$$\tilde{\sum_{p=1,2...l}^{+}} \tilde{\sum_{i=1,2...m}^{+}} \tilde{\sum_{j=1,2...n}^{+}} \tilde{\sum_{k=1,2...o}^{+}} \tilde{c}^{tp}_{ijk} \otimes \tilde{x}^{p}_{ijk}, t = 1,2,3.....R$$

Theorem 1:

The integration of optimal solution of four sub- problems (4)-(7) provides the optimal solution of the FFMOMISTP (3).

Proof: Let $\tilde{\bar{x}}^{p*}_{ijk} = \left(\bar{x}^{p*}_{ijk,1}, \bar{x}^{p*}_{ijk,2}, \bar{x}^{p*}_{ijk,3}, \bar{x}^{p*}_{ijk,4}\right)_{LR}$ be an arbitrary fuzzy feasible solution of FFMOMISTP (3) it follows that

$$(i)\sum_{j=1}^{n}\sum_{k=1}^{o}\bar{x}^{p*}_{ijk,1} = s^{p}_{i,1}\ i = 1,2,...m, p = 1,2...l,\ \sum_{i=1}^{m}\sum_{k=1}^{o}\bar{x}^{p*}_{ijk,1} = d^{p}_{j,1},\ j = 1,2...n, \mathrm{p} = 1,2...l$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\bar{x}^{p*}_{ijk,1} = e_{k,1}\ k = 1,2...o,\ \bar{x}^{p*}_{ijk,1}\ 3\ 0, i = 1,2...m, j = 1,2...n, k = 1,2...o, p = 1,2...l$$

$$(ii)\sum_{j=1}^{n}\sum_{k=1}^{o}\bar{x}^{p*}_{ijk,2} = s^{p}_{i,2}\ i = 1,2,...m, p = 1,2...l,\ \sum_{i=1}^{m}\sum_{k=1}^{o}\bar{x}^{p*}_{ijk,2} = d^{p}_{j,2},\ j = 1,2...n, \mathrm{p} = 1,2...l$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\bar{x}^{p*}_{ijk,2} = e_{k,2}\ k = 1,2...o,\ \bar{x}^{p*}_{ijk,2}\ 3\ \bar{x}^{p*}_{ijk,1}, i = 1,2...m, j = 1,2...n, k = 1,2...o, p = 1,2...l$$

$$(iii)\sum_{j=1}^{n}\sum_{k=1}^{o}\bar{x}^{p*}_{ijk,3} = s^{p}_{i,3}\ i = 1,2,...m, p = 1,2...l,\ \sum_{i=1}^{m}\sum_{k=1}^{o}\bar{x}^{p*}_{ijk,3} = d^{p}_{j,3},\ j = 1,2...n, \mathrm{p} = 1,2...l$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\bar{x}^{p*}_{ijk,3} = e_{k,3}\ k = 1,2...o,\ \bar{x}^{p*}_{ijk,3}\ 3\ \bar{x}^{p*}_{ijk,2}, i = 1,2...m, j = 1,2...n, k = 1,2...o, p = 1,2...l$$

$$(iv)\sum_{j=1}^{n}\sum_{k=1}^{o}\bar{x}^{p*}_{ijk,4} = s^{p}_{i,4}\ i = 1,2,...m, p = 1,2...l,\ \sum_{i=1}^{m}\sum_{k=1}^{o}\bar{x}^{p*}_{ijk,4} = d^{p}_{j,4},\ j = 1,2...n, \mathrm{p} = 1,2...l$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\bar{x}^{p*}_{ijk,4} = e_{k,4}\ k = 1,2...o,\ \bar{x}^{p*}_{ijk,4}\ 3\ \bar{x}^{p*}_{ijk,3}, i = 1,2...m, j = 1,2...n, k = 1,2...o, p = 1,2...l$$

The Condition (i), (ii), (iii) and (iv) imply that $\bar{x}^{p*}_{ijk,1}, \bar{x}^{p*}_{ijk,2}, \bar{x}^{p*}_{ijk,3}$ *and* $\bar{x}^{p*}_{ijk,4}$ are the feasible solutions of problems (4), (5), (6) and (7) respectively, Also, due to optimality of $x^{p*}_{ijk,1}, x^{p*}_{ijk,2}, x^{p*}_{ijk,3}$ and $x^{p*}_{ijk,4}$ respectively for problems (4), (5), (6), and (7), we conclude that

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,1}^{tp}\, x_{ijk,1}^{p*} \leq \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,1}^{tp}\, \overline{x}_{ijk,1}^{p*}$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,2}^{tp}\, x_{ijk,2}^{p*} \leq \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,2}^{tp}\, \overline{x}_{ijk,2}^{p*}$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,3}^{tp}\, x_{ijk,3}^{p*} \leq \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,3}^{tp}\, \overline{x}_{ijk,3}^{p*}$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,4}^{tp}\, x_{ijk,4}^{p*} \leq \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,4}^{tp}\, \overline{x}_{ijk,4}^{p*}$$

Regarding to definition 2.7, this means that

$$\left[\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,1}^{tp}\, x_{ijk,1}^{p*}, \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,2}^{tp}\, x_{ijk,2}^{p*}, \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,3}^{tp}\, x_{ijk,3}^{p*}, \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,4}^{tp}\, x_{ijk,4}^{p*}\right]_{LR}$$

$$\tilde{\leq}\left[\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,1}^{tp}\, \overline{x}_{ijk,1}^{p*}, \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,2}^{tp}\, \overline{x}_{ijk,2}^{p*}, \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,3}^{tp}\, \overline{x}_{ijk,3}^{p*}, \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{0} c_{ijk,4}^{tp}\, \overline{x}_{ijk,4}^{p*}\right]_{LR}$$ Therefore

$\tilde{\sum}_{p=1,2...l}\tilde{\sum}_{i=1,2...m}\tilde{\sum}_{j=1,2...n}\tilde{\sum}_{k=1,2...o} \tilde{c}_{ijk}^{tp} \otimes \tilde{x}_{ijk}^{p*} \tilde{\leq} \tilde{\sum}_{p=1,2...l}\tilde{\sum}_{i=1,2...m}\tilde{\sum}_{j=1,2...n}\tilde{\sum}_{k=1,2...o} \tilde{c}_{ijk}^{tp} \otimes \overline{\tilde{x}}_{ijk}^{p*}$ and hence $\tilde{x}_{ijkp}^{*} = \left(x_{ijkp,1}^{*}, x_{ijkp,2}^{*}, x_{ijkp,3}^{*}, x_{ijkp,4}^{*}\right)_{LR.}$ will be the fuzzy optimal solution of FFMOMISTP(3) for each objective.

Finally the obtained fuzzy optimal solution is in the form of a non-negative LR flat fuzzy number. The optimal solution for FFMOMISTP can be obtain by goal programming approach

4.5 Fuzzy goal programming approach for solving FFMOMISTP**:**

The procedure for fuzzy goal programming approach as follows:

According to Zangiabadi and H.R.Maleki (2013) ''

Step: 1 Solve the multi objective transportation problem as a single objective transportation problem, taking each time only one objective as objective function and ignoring all others.

Step: 2 Compute the value of each objective function at each solution derived in Step1.

Step: 3 From Step,2find for each objective the best (L_r) and the worst (U_r) values corresponding to the set of solutions. Recall that L_r and U_r are the aspired level of achievement and the highest acceptable level of achievement for the r -th objective function respectively.

Step: 4

The membership functions μ_r for ther^{th} objective function is defined as follows:

$$\mu\,(Z_r\,(x)\,) = \begin{cases} 1 & ,\text{if } Z_r \leq L_r \\ 1-\dfrac{Z_r - L_r}{U_r - L_r} & ,\text{if } L_r < Z_r < U_r \\ 0 & ,\text{if } Z_r \geq U_r \end{cases}$$

then an equivalent linear model can be formulated as:

min :λ

S.t.

$$\frac{U_r - Z_r}{U_r - L_r} + d_r^- - d_r^+ = 1,$$

$\phi \geq d_r^-$, r = 1, 2, …..k,

$d_r^+ d_r^- = 0,$

$$\sum_{j=1,2,\ldots,n}^{\tilde{+}} \sum_{k=1,2\ldots o}^{\tilde{+}} \tilde{x}_{ijk}^{p} = \tilde{s}_i^{p}, i = 1,2,\ldots m, p = 1,2\ldots l$$

$$\sum_{i=1,2\ldots m}^{\tilde{+}} \sum_{k=1,2\ldots o}^{\tilde{+}} \tilde{x}_{ijk}^{p} = \tilde{d}_j^{p}, \quad j = 1,2\ldots n, p = 1,2\ldots l$$

$$\sum_{p=1,2\ldots l}^{\tilde{+}} \sum_{i=1,2\ldots m}^{\tilde{+}} \sum_{j=1,2\ldots n}^{\tilde{+}} \tilde{x}_{ijk}^{p} = \tilde{e}_k, \quad k = 1,2\ldots o$$

$\tilde{x}_{ijk}^{p} \in \ell\Re(\ )^+$, $i = 1,2\ldots m$, $j = 1,2\ldots n$, $k = 1,2\ldots o$, $p = 1,2\ldots l$

$d_r^+, d_r^- \geq 0$,

$\phi \leq 1$, $\phi \geq 0$, $x_{ij} \geq 0$, for all i, j.

where the equilibrium condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

Step: 5

Solve the equivalent crisp model obtained in step 4. The solution obtained is step 5 will be the optimal compromise solution of FFMOMISTP. ''

5. Numerical Example:

A fruit supplier company of west Bengal, India supply two types of fruits namely mango and orange from three source points namely Darjeeling, siligiri and Malda of west Bengal, India by truck, and lorry to the four destinations situated at Kolkata, Ranchi, Durgapur and Kharagpur through twelve routes. Our objective is to minimize the transportation cost ,packaging cost and emission cost related to fuels and packaging. Two models have been solved model 1 is related to non-eco-friendly fuel and packaging and model-2 is related to eco-friendly fuel and packaging . Due to fluctuation of fuel price, road tax for different route, political issues, different type of packaging for different route, fluctuation of price packaging material the transportation cost and packaging cost in each route is not fixed. The transportation cost and packaging cost of carrying one unit (500 gm) of mango and orange form source to a destination is treated as a multi objective multi- item solid transportation problem. Here we have considered the parameters of transportation problem in LR flat fuzzy environment. The input data are given as follows:

Model-1Transportation cost for 1st item

$$\begin{bmatrix} C_{1111} & C_{1211} & C_{1311}\,C_{1411} & C_{2111} & C_{2211} \\ C_{2311} & C_{2411} & C_{3111}\,C_{3211} & C_{3311} & C_{3411} \\ C_{1121} & C_{1221} & C_{1321}\,C_{1421} & C_{2121} & C_{2221} \\ C_{2321} & C_{2421} & C_{3121}\,C_{3221} & C_{3321} & C_{3421} \end{bmatrix} = \begin{bmatrix} (.5,.51,.52,.54) & (.39,.4,.42,.43) & (.51,.53,.54,.55)\,(.47,.49,.5,.51) & (.35,.36,.38,.39) & (.48,.49,.51,.53) \\ (.35,.36,.38,.4) & (.18,.19,.21,.22) & (.30,.31,.32,.34)\,(.68,.69,.71,.72) & (.83,.84,.86,.87) & (.73,.75,.76,.77) \\ (.5,.51,.52,.54) & (.38,.39,.42,.43) & (.5,.51,.53,.55)\,(.46,.48,.49,.51) & (.34,.35,.37,.38) & (.47,.48,.5,.52) \\ (.34,.35,.36,.38) & (.18,.19,.22,.24) & (.29,.30,.31,.32)\,(.67,.68,.7,.72) & (.82,.83,.85,.87) & (.72,.73,.75,.77) \end{bmatrix}$$

Transportation cost for 2nd item.

$$\begin{bmatrix} C_{1112} & C_{1212} & C_{1312}\,C_{1412} & C_{2112} & C_{2212} \\ C_{2312} & C_{2412} & C_{3112}\,C_{3212} & C_{3312} & C_{3412} \\ C_{1122} & C_{1222} & C_{1322}\,C_{1422} & C_{2121} & C_{2222} \\ C_{2322} & C_{2422} & C_{3122}\,C_{3222} & C_{3322} & C_{3422} \end{bmatrix} = \begin{bmatrix} (.45,.46,.47,.49) & (.36,.37,.39,.40) & (.49,.51,.52,.54) & (.44,.45,.46,.48) & (.35,.37,.39,.41) & (.5,.52,.53,.55) \\ (.35,.36,.37,.39) & (.32,.33,.34,.36) & (.28,.3,.31,.33) & (.62,.63,.65,.68) & (.78,.79,.80,.81) & (.69,.70,.72,.73) \\ (.44,.45,.47,.48) & (.35,.36,.38,.39) & (.48,.49,.50,.52) & (.43,.44,.46,.47) & (.34,.35,.37,.38) & (.49,.5,.51,.53) \\ (.34,.35,.36,.38) & (.3,.32,.33,.35) & (.25,.27,.28,.29) & (.60,.61,.63,.64) & (.76,.77,.78,.79) & (.66,.67,.68,.69) \end{bmatrix}$$

Packaging cost for 1st item

$$\begin{bmatrix} C_{1111} & C_{1211} & C_{1311}\,C_{1411} & C_{2111} & C_{2211} \\ C_{2311} & C_{2411} & C_{3111}\,C_{3211} & C_{3311} & C_{3411} \\ C_{1121} & C_{1221} & C_{1321}\,C_{1421} & C_{2121} & C_{2221} \\ C_{2321} & C_{2421} & C_{3121}\,C_{3221} & C_{3321} & C_{3421} \end{bmatrix} = \begin{bmatrix} (.23,.24,.26,.27) & (.25,.26,.27,.28) & (.27,.28,.29,.3) & (.28,.27,.29,.30) & (.26,.27,.28,.3) & (.26,.27,.28,.3) \\ (.4,.41,.42,.44) & (.3,.31,.33,.35) & (.25,.27,.28,.31) & (.27,.28,.29,.3) & (.4,.41,.43,.44) & (.3,.32,.33,.34) \\ (.23,.24,.26,.27) & (.25,.26,.27,.28) & (.27,.28,.29,.3) & (.28,.27,.29,.30) & (.26,.27,.28,.3) & (.26,.27,.28,.3) \\ (.4,.41,.42,.44) & (.3,.31,.33,.35) & (.25,.27,.28,.31) & (.27,.28,.29,.3) & (.4,.41,.43,.44) & (.3,.32,.33,.34) \end{bmatrix}$$

Packaging cost for 2nd item

$$\begin{bmatrix} C_{1112} & C_{1212} & C_{1312}\,C_{1412} & C_{2112} & C_{2212} \\ C_{2312} & C_{2412} & C_{3112}\,C_{3212} & C_{3312} & C_{3412} \\ C_{1122} & C_{1222} & C_{1322}\,C_{1422} & C_{2121} & C_{2222} \\ C_{2322} & C_{2422} & C_{3122}\,C_{3222} & C_{3322} & C_{3422} \end{bmatrix} = \begin{bmatrix} (.23,.24,.25,.27) & (.25,.26,.28,.30) & (.25,.26,.28,.3) & (.27,.28,.29,.31) & (.26,.27,.29,.31) & (.3,.31,.32,.34) \\ (.35,.36,.37,.39) & (.32,.33,.35,.36) & (.25,.26,.27,.29) & (.26,.27,.29,.30) & (.26,.27,.29,.30) & (.28,.29,.3,.31) \\ (.23,.24,.25,.27) & (.25,.26,.28,.30) & (.25,.26,.28,.3) & (.27,.28,.29,.31) & (.26,.27,.29,.31) & (.3,.31,.32,.34) \\ (.35,.36,.37,.39) & (.32,.33,.35,.36) & (.25,.26,.27,.29) & (.26,.27,.29,.30) & (.26,.27,.29,.30) & (.28,.29,.3,.31) \end{bmatrix}$$

Emission cost for 1st item

$$\begin{bmatrix} C_{1111} & C_{1211} & C_{1311}\,C_{1411} & C_{2111} & C_{2211} \\ C_{2311} & C_{2411} & C_{3111}\,C_{3211} & C_{3311} & C_{3411} \\ C_{1121} & C_{1221} & C_{1321}\,C_{1421} & C_{2121} & C_{2221} \\ C_{2321} & C_{2421} & C_{3121}\,C_{3221} & C_{3321} & C_{3421} \end{bmatrix} = \begin{bmatrix} (.023,.025,.026,.027) & (.018,.019,.02,.021) & (.024,.025,.026,.028) & (.021,.022,.024,.025) & (.016,.017,.019,.020) & (.022,.023,.025,.026) \\ (.015,.016,.018,.019) & (.008,.009,.010,.011) & (.014,.015,.016,.018) & (.031,.032,.034,.035) & (.037,.038,.039,.04) & (.033,.034,.036,.037) \\ (.026,.027,.028,.029) & (.019,.02,.021,.023) & (.027,.028,.029,.03) & (.024,.025,.027,.028) & (.018,.019,.02,.021) & (.024,.025,.027,.028) \\ (.018,.019,.02,.021) & (.009,.010,.011,.012) & (.016,.017,.018,.019) & (.035,.036,.038,.04) & (.043,.044,.045,.046) & (.038,.039,.04,.041) \end{bmatrix}$$

Emission cost for 2nd item

$$\begin{bmatrix} C_{1112} & C_{1212} & C_{1312}\,C_{1412} & C_{2112} & C_{2212} \\ C_{2312} & C_{2412} & C_{3112}\,C_{3212} & C_{3312} & C_{3412} \\ C_{1122} & C_{1222} & C_{1322}\,C_{1422} & C_{2121} & C_{2222} \\ C_{2322} & C_{2422} & C_{3122}\,C_{3222} & C_{3322} & C_{3422} \end{bmatrix} = \begin{bmatrix} (.021,.022,.023,.025) & (.017,.018,.019,.020) & (.022,.023,.025,.026) & (.019,.02,.022,.023) & (.016,.018,.019,.020) & (.024,.025,.026,.027) \\ (.015,.016,.018,.019) & (.011,.012,.013,.014) & (.012,.013,.014,.016) & (.029,.03,.031,.032) & (.035,.036,.037,.038) & (.03,.031,.032,.033) \\ (.024,.025,.026,.028) & (.017,.018,.02,.021) & (.025,.026,.027,.03) & (.022,.023,.024,.026) & (.018,.019,.02,.021) & (.026,.027,.028,.03) \\ (.018,.019,.020,.021) & (.013,.015,.016,.018) & (.026,.027,.028,.029) & (.033,.034,.035,.036) & (.042,.043,.044,.046) & (.035,.036,.037,.040) \end{bmatrix}$$

Amounts of items available at origin

$$\begin{bmatrix} S_i^{(p)} & S_i^{(p)} & S_i^{(p)} \\ S_i^{(p)} & S_i^{(p)} & S_i^{(p)} \end{bmatrix} = \begin{bmatrix} (600,800,900,1100) & (100,400,600,700) & (1000,1100,1300,1500) \\ (700,900,1000,1200) & (200,300,500,700) & (700,900,1100,1200) \end{bmatrix}$$

The demand amount of items at destination

$$\begin{bmatrix} d_j^{(p)} & d_j^{(p)} & d_j^{(p)} & d_j^{(p)} \\ d_j^{(p)} & d_j^{(p)} & d_j^{(p)} & d_j^{(p)} \end{bmatrix} = \begin{bmatrix} (700,900,1000,1100) & (500,700,800,900) & (300,400,500,600) & (200,300,500,700) \\ (600,700,800,1000) & (600,700,800,900) & (300,500,600,700) & (100,200,400,500) \end{bmatrix}$$

Conveyance capacity.

$$\begin{bmatrix} e_k & e_k \end{bmatrix} = \begin{bmatrix} (2000,2500,3000,3500) & (1300,1900,2400,2900) \end{bmatrix}$$

Using the solution methodology prescribed in sec.4,the total fuzzy transportation cost of model 1 is as follows:

$$Z_1^* = \sum_{p=1}^{2}\sum_{i=1}^{3}\sum_{j=1}^{4}\sum_{k=1}^{2} \tilde{c}_{ijk}^{p} \otimes \tilde{x}_{ijk}^{p} = (1252,1711,2177,2667)_{LR} \quad Z_2^* = \sum_{p=1}^{2}\sum_{i=1}^{3}\sum_{j=1}^{4}\sum_{k=1}^{2} \tilde{c}_{ijk}^{p} \otimes \tilde{x}_{ijk}^{p} = (847,1187,1547,1947)_{LR}$$

$$Z_3^* = \sum_{p=1}^{2}\sum_{i=1}^{3}\sum_{j=1}^{4}\sum_{k=1}^{2} \tilde{c}_{ijk}^{p} \otimes \tilde{x}_{ijk}^{p} = (59.1,82.7,108.2,137.5)_{LR}$$

The fuzzy optimal solution and total fuzzy transportation cost based on this proposed method have no negative part.

Now, we have formulated the following membership functions of model 1 is as follows:

$$\mu(z_1^1(x)) = \begin{cases} 0, & if Z_1^1(x) \geq U_1^1 \\ \dfrac{1569 - Z_1^1(x)}{1569-1252}, & if\ L_1^1 < Z_1^1(x) < U_1^1 \\ 1, & if Z_1^1(x) \leq L_1^1 \end{cases}, \quad \mu(z_1^2(x)) = \begin{cases} 0, & if Z_1^2(x) \geq U_1^2 \\ \dfrac{2139 - Z_1^2(x)}{2139-1711}, & if\ L_1^2 < Z_1^2(x) < U_1^2 \\ 1, & if Z_1^2(x) \leq L_1^2 \end{cases}, \quad \mu(z_1^3(x)) = \begin{cases} 0, & if Z_1^3(x) \geq U_1^3 \\ \dfrac{2724 - Z_1^3(x)}{2724-2177}, & if\ L_1^3 < Z_1^3(x) < U_1^3 \\ 1, & if Z_1^3(x) \leq L_1^3 \end{cases}$$

$$\mu(z_1^4(x)) = \begin{cases} 0, & if Z_1^4(x) \geq U_1^4 \\ \dfrac{3344 - Z_1^4(x)}{3344-2667}, & if\ L_1^4 < Z_1^4(x) < U_1^4 \\ 1, & if Z_1^4(x) \leq L_1^4 \end{cases}, \quad \mu(z_2^1(x)) = \begin{cases} 0, & if Z_2^1(x) \geq U_2^1 \\ \dfrac{89 - Z_2^1(x)}{89-847}, & if\ L_2^1 < Z_2^1(x) < U_2^1 \\ 1, & if Z_2^1(x) \leq L_2^1 \end{cases}, \quad \mu(z_2^2(x)) = \begin{cases} 0, & if Z_2^2(x) \geq U_2^2 \\ \dfrac{1258 - Z_2^1(x)}{1258-1187}, & if\ L_2^2 < Z_2^2(x) < U_2^2 \\ 1, & if Z_2^2(x) \leq L_2^2 \end{cases}$$

$$\mu(z_2^3(x))=\begin{cases}0, & if\,Z_2^3(x)\geq U_2^3\\ \frac{1627-Z_2^1(x)}{1627-1547}, & if\ L_2^3<Z_2^3(x)<U_2^3\\ 1, & if\,Z_2^3(x)\leq L_2^3\end{cases},\quad \mu(z_2^4(x))=\begin{cases}0, & if\,Z_2^4(x)\geq U_2^4\\ \frac{2045-Z_2^4(x)}{2045-1947}, & if\ L_2^4<Z_2^4(x)<U_2^4\\ 1, & if\,Z_2^4(x)\leq L_2^4\end{cases},\quad \mu(z_3^1(x))=\begin{cases}0, & if\,Z_3^1(x)\geq U_3^1\\ \frac{77.8-Z_3^1(x)}{77.8-59.1}, & if\ L_3^1<Z_3^1(x)<U_3^1\\ 1, & if\,Z_3^1(x)\leq L_3^1\end{cases}$$

$$\mu(z_3^2(x))=\begin{cases}0, & if\,Z_3^2(x)\geq U_3^2\\ \frac{109-Z_3^2(x)}{109-82.7}, & if\ L_3^2<Z_3^2(x)<U_3^2\\ 1, & if\,Z_3^2(x)\leq L_3^2\end{cases},\quad \mu(z_3^3(x))=\begin{cases}0, & if\,Z_3^3(x)\geq U_3^3\\ \frac{140.4-Z_3^3(x)}{140.4-108.2}, & if\ L_3^3<Z_3^3(x)<U_3^3\\ 1, & if\,Z_3^3(x)\leq L_3^3\end{cases},\quad \mu(z_3^4(x))=\begin{cases}0, & if\,Z_3^4(x)\geq U_3^4\\ \frac{177.3-Z_3^4(x)}{177.3-137.5}, & if\ L_3^4<Z_3^4(x)<U_3^4\\ 1, & if\,Z_3^4(x)\leq L_3^4\end{cases}$$

The fuzzy optimal solution of model 1 using goal programming method is as follows:

$\tilde{x}^*_{1111}=(0,\ 60,\ 60,\ 60)_{LR}$	$\tilde{x}^*_{1211}=(300,\ 338,\ 338,\ 338)_{LR}$,	$\tilde{x}^*_{1311}=(0,\ 100,\ 167,\ 267)_{LR}$,	$\tilde{x}^*_{1411}=(0,\ 0,\ 0,\ 0)_{LR}$,
$\tilde{x}^*_{2111}=(0,\ 0,\ 5,\ 5)_{LR}$,	$\tilde{x}^*_{2211}=(0,\ 0,8,\ 8)_{LR}$,	$\tilde{x}^*_{2311}=(0,\ 0,\ 0,\ 0)_{LR}$,	$\tilde{x}^*_{2111}=(0,\ 0,\ 5,\ 5)_{LR}$,
$\tilde{x}^*_{3111}=(700,\ 700,\ 753,\ 847)_{LR}$	$\tilde{x}^*_{3211}=(200,\ 200,\ 200,\ 300)_{LR}$	$\tilde{x}^*_{3311}=(0,\ 0,\ 0,\ 0)_{LR}$	$\tilde{x}^*_{3411}=(100,\ 100,\ 246,\ 252)_{LR}$
$\tilde{x}^*_{1122}=(0,\ 0,\ 0,\ 0)_{LR}$	$\tilde{x}^*_{1222}=(400,\ 400,\ 400,\ 500)_{LR}$	$\tilde{x}^*_{1322}=(155,\ 355,\ 355,\ 355)_{LR}$	$\tilde{x}^*_{1422}=(0,\ 0,\ 7,\ 7)_{LR}$
$\tilde{x}^*_{2122}=(57,\ 57,\ 57,\ 257)_{LR}$	$\tilde{x}^*_{2322}=(0,\ 0,\ 0,\ 0)_{LR}$	$\tilde{x}^*_{2422}=(100,\ 100,\ 171,\ 171)_{LR}$	$\tilde{x}^*_{2222}=(43,\ 43,\ 131,\ 131)_{LR}$
$\tilde{x}^*_{3122}=(145,\ 145,\ 245,\ 245)_{LR}$	$\tilde{x}^*_{3222}=(0,\ 0,\ 11,\ 11)_{LR}$	$\tilde{x}^*_{3322}=(0,\ 0,\ 9,\ 9)_{LR}$	$\tilde{x}^*_{3422}=(0,\ 0,\ 12,\ 12)_{LR}$
$\tilde{x}^*_{1121}=(0,\ 0,\ 1,\ 1)_{LR}$	$\tilde{x}^*_{1221}=(0,\ 1,\ 1,\ 1)_{LR}$	$\tilde{x}^*_{1321}=(300,\ 300,\ 332,\ 332)_{LR}$	$\tilde{x}^*_{1421}=(0,\ 1,\ 1,\ 101)_{LR}$
$\tilde{x}^*_{2121}=(0,\ 40,\ 81,\ 87)_{LR}$	$\tilde{x}^*_{2221}=(0,\ 161,\ 253,\ 253)_{LR}$	$\tilde{x}^*_{2321}=(0,\ 0,\ 1,\ 1)_{LR}$	$\tilde{x}^*_{2421}=(100,\ 197,\ 231,\ 325)_{LR}$
$\tilde{x}^*_{3121}=(0,\ 100,\ 100,\ 200)_{LR}$	$\tilde{x}^*_{3221}=(0,\ 0,\ 0,\ 0)_{LR}$	$\tilde{x}^*_{3321}=(0,\ 0,\ 0,\ 0)_{LR}$	$\tilde{x}^*_{3321}=(0,\ 0,\ 0,\ 0)_{LR}$
$\tilde{x}^*_{1112}=(0,\ 0,\ 0,\ 0)_{LR}$	$\tilde{x}^*_{1212}=(0,\ 0,\ 0,\ 0)_{LR}$	$\tilde{x}^*_{1312}=(145,\ 145,\ 235,\ 335)_{LR}$	$\tilde{x}^*_{1412}=(0,\ 0,\ 3,\ 3)_{LR}$
$\tilde{x}^*_{2112}=(0,\ 0,\ 0,\ 0)_{LR}$	$\tilde{x}^*_{2212}=(0,\ 0,\ 1,\ 1)_{LR}$	$\tilde{x}^*_{2312}=(0,\ 0,\ 1,\ 1)_{LR}$	$\tilde{x}^*_{2412}=(0,\ 100,\ 139,\ 139)_{LR}$
$\tilde{x}^*_{3112}=(398,\ 498,\ 498,\ 498)_{LR}$	$\tilde{x}^*_{3212}=(157,\ 257,\ 257,\ 257)_{LR}$	$\tilde{x}^*_{3312}=(0,\ 0,\ 0,\ 0)_{LR}$	$\tilde{x}^*_{3412}=(0,\ 0,\ 68,\ 168)_{LR}$

$\lambda=(.1601,\ .3142,\ .3142,\ .4343)_{LR}$, $d_1^-=(0.1597,\ 0.1597,\ 0.313,\ .3908)_{LR}$, $d_2^-=(0.1601,\ 0.237,\ 0.2553,\ 0.4343)_{LR}$, $d_3^-=(0.16,\ 0.3142,\ 0.3142,\ 0.4341)_{LR}$,
$z_1=(1304.93,1810.57,2370.33,2942.64)_{LR}$, $z_2=(856.29,1200.06,1563.83,1969.43)_{LR}$,
$z_3=(64.108,.90.448,120.156,154.055)_{LR}$

Model-2

Transportation cost for 1st item

$$\begin{bmatrix}C_{1111} & C_{1211} & C_{1311} & C_{1411} & C_{2111} & C_{2211}\\ C_{2311} & C_{2411} & C_{3111} & C_{3211} & C_{3311} & C_{3411}\\ C_{1121} & C_{1221} & C_{1321} & C_{1421} & C_{2121} & C_{2221}\\ C_{2321} & C_{2421} & C_{3121} & C_{3221} & C_{3321} & C_{3421}\end{bmatrix}=\begin{bmatrix}(.52,.53,.54,.55) & (.40,.41,.43,.45) & (.53,.55,.56,.58) & (.48,.49,.51,.53) & (.36,.38,.39,.4) & (.5,.51,.53,.55)\\ (.36,.37,.38,.39) & (.2,.22,.23,.25) & (.32,.33,.34,.36) & (.70,.72,.73,.75) & (.86,.88,.89,.9) & (.76,.77,.78,.79)\\ (.52,.53,.55,.56) & (.39,.4,.42,.43) & (.52,.53,.54,.55) & (.48,.49,.5,.52) & (.36,.37,.38,.39) & (.49,.51,.53,.54)\\ (.35,.36,.37,.39) & (.19,.2,.22,.24) & (.31,.33,.35,.37) & (.69,.70,.72,.74) & (.84,.85,.86,.88) & (.74,.75,.76,.77)\end{bmatrix}$$

Transportation cost for 2nd item

$$\begin{bmatrix}C_{1112} & C_{1212} & C_{1312} & C_{1412} & C_{2112} & C_{2212}\\ C_{2312} & C_{2412} & C_{3112} & C_{3212} & C_{3312} & C_{3412}\\ C_{1122} & C_{1222} & C_{1322} & C_{1422} & C_{2121} & C_{2222}\\ C_{2322} & C_{2422} & C_{3122} & C_{3222} & C_{3322} & C_{3422}\end{bmatrix}=\begin{bmatrix}(.47,.48,.49,.50) & (.38,.39,.41,.43) & (.51,.53,.55,.57) & (.46,.47,.49,.51) & (.36,.38,.39,.40) & (.52,.53,.54,.55)\\ (.36,.37,.39,.41) & (.25,.26,.27,.29) & (.30,.31,.33,.35) & (.65,.66,.68,.70) & (.84,.85,.86,.87) & (.70,.72,.73,.75)\\ (.45,.46,.47,.49) & (.37,.38,.39,.40) & (.49,.5,.51,.53) & (.44,.45,.47,.48) & (.36,.37,.39,.41) & (.51,.52,.53,.55)\\ (.35,.37,.38,.40) & (.23,.24,.26,.27) & (.28,.29,.31,.33) & (.63,.65,.66,.67) & (.82,.83,.84,.86) & (.69,.70,.72,.73)\end{bmatrix}$$

Bio packaging cost for 1st item

$$\begin{bmatrix}C_{1111} & C_{1211} & C_{1311} & C_{1411} & C_{2111} & C_{2211}\\ C_{2311} & C_{2411} & C_{3111} & C_{3211} & C_{3311} & C_{3411}\\ C_{1121} & C_{1221} & C_{1321} & C_{1421} & C_{2121} & C_{2221}\\ C_{2321} & C_{2421} & C_{3121} & C_{3221} & C_{3321} & C_{3421}\end{bmatrix}=\begin{bmatrix}(1.33,1.34,1.35,1.37) & (1.35,1.36,1.38,1.39) & (1.37,1.38,1.39,1.41) & (1.42,1.43,1.44,1.46) & (1.36,1.37,1.38,1.4) & (1.36,1.37,1.38,1.4)\\ (1.45,1.46,1.47,1.49) & (1.4,1.42,1.43,1.44) & (1.35,1.36,1.38,1.40) & (1.37,1.38,1.41,1.43) & (1.45,1.46,1.47,1.49) & (1.4,1.42,1.43,1.44)\\ (1.33,1.34,1.35,1.37) & (1.35,1.36,1.38,1.39) & (1.37,1.38,1.39,1.41) & (1.42,1.43,1.44,1.46) & (1.36,1.37,1.38,1.4) & (1.36,1.37,1.38,1.4)\\ (1.45,1.46,1.47,1.49) & (1.4,1.42,1.43,1.44) & (1.35,1.36,1.38,1.40) & (1.37,1.38,1.41,1.43) & (1.45,1.46,1.47,1.49) & (1.4,1.42,1.43,1.44)\end{bmatrix}$$

Bio packaging cost for 2nd item

$$\begin{bmatrix} C_{1112} & C_{1212} & C_{1312} & C_{1412} & C_{2112} & C_{2212} \\ C_{2312} & C_{2412} & C_{3112} & C_{3212} & C_{3312} & C_{3412} \\ C_{1122} & C_{1222} & C_{1322} & C_{1422} & C_{2121} & C_{2222} \\ C_{2322} & C_{2422} & C_{3122} & C_{3222} & C_{3322} & C_{3422} \end{bmatrix} = \begin{bmatrix} (1.33,1.34,1.35,1.37) & (1.35,1.36,1.38,1.39) & (1.35,1.36,1.38,1.39) & (1.37,1.38,1.39,1.40) & (1.36,1.38,1.40,1.41) & (1.4,1.41,1.43,1.45) \\ (1.41,1.43,1.44,1.45) & (1.42,1.43,1.44,1.46) & (1.35,1.36,1.38,1.40) & (1.36,1.38,1.40,1.41) & (1.36,1.38,1.40,1.41) & (1.37,1.38,1.41,1.42) \\ (1.33,1.34,1.35,1.37) & (1.35,1.36,1.38,1.39) & (1.35,1.36,1.38,1.39) & (1.37,1.38,1.39,1.40) & (1.36,1.38,1.40,1.41) & (1.4,1.41,1.43,1.45) \\ (1.41,1.43,1.44,1.45) & (1.42,1.43,1.44,1.46) & (1.35,1.36,1.38,1.40) & (1.36,1.38,1.40,1.41) & (1.36,1.38,1.40,1.41) & (1.37,1.38,1.41,1.42) \end{bmatrix}$$

Emission cost for 1stitem with Biodiesel.

$$\begin{bmatrix} C_{1111} & C_{1211} & C_{1311} & C_{1411} & C_{2111} & C_{2211} \\ C_{2311} & C_{2411} & C_{3111} & C_{3211} & C_{3311} & C_{3411} \\ C_{1121} & C_{1221} & C_{1321} & C_{1421} & C_{2121} & C_{2221} \\ C_{2321} & C_{2421} & C_{3121} & C_{3221} & C_{3321} & C_{3421} \end{bmatrix} = \begin{bmatrix} (.006,.007,.008,.009) & (.005,.006,.007,.008) & (.006,.008,.009,.010) & (.005,.006,.009,.010) & (.004,.006,.007,.009) & (.006,.008,.009,.010) \\ (.004,.005,.007,.008) & (.002,.003,.005,.006) & (.003,.004,.006,.007) & (.008,.009,.010,.011) & (.009,.010,.011,.013) & (.008,.009,.010,.013) \\ (.007,.008,.009,.010) & (.005,.006,.008,.009) & (.007,.008,.009,.010) & (.006,.007,.009,.010) & (.004,.005,.007,.008) & (.007,.008,.009,.010) \\ (.005,.006,.008,.010) & (.002,.003,.005,.007) & (.004,.005,.006,.009) & (.009,.010,.012,.013) & (.011,.013,.014,.016) & (.010,.012,.013,.014) \end{bmatrix}$$

Emission cost for 2nd item with Biodiesel.

$$\begin{bmatrix} C_{1112} & C_{1212} & C_{1312} & C_{1412} & C_{2112} & C_{2212} \\ C_{2312} & C_{2412} & C_{3112} & C_{3212} & C_{3312} & C_{3412} \\ C_{1122} & C_{1222} & C_{1322} & C_{1422} & C_{2121} & C_{2222} \\ C_{2322} & C_{2422} & C_{3122} & C_{3222} & C_{3322} & C_{3422} \end{bmatrix} = \begin{bmatrix} (.004,.005,.007,.008) & (.004,.005,.007,.008) & (.005,.007,.009,.010) & (.003,.004,.006,.008) & (.004,.005,.007,.008) & (.007,.009,.010,.011) \\ (.004,.005,.007,.008) & (.003,.004,.006,.008) & (.001,.002,.004,.005) & (.006,.007,.009,.010) & (.007,.008,.009,.010) & (.006,.007,.009,.010) \\ (.005,.007,.009,.010) & (.004,.005,.006,.007) & (.006,.007,.008,.009) & (.005,.007,.009,.010) & (.004,.005,.006,.007) & (.008,.009,.010,.011) \\ (.005,.007,.009,.010) & (.003,.004,.005,.007) & (.002,.003,.005,.007) & (.007,.008,.009,.010) & (.009,.010,.011,.012) & (.008,.009,.010,.011) \end{bmatrix}$$

Using the solution methodology prescribed in sec.4, the total fuzzy transportation cost of model 2 is as follows:

$$z_1^* = \sum_{p=1}^{2}\sum_{i=1}^{3}\sum_{j=1}^{4}\sum_{k=1}^{2} \tilde{c}_{ijk}^{p} \otimes \tilde{x}_{ijk}^{p} = (1313,1781,2253,2779)_{LR} \quad z_2^* = \sum_{p=1}^{2}\sum_{i=1}^{3}\sum_{j=1}^{4}\sum_{k=1}^{2} \tilde{c}_{ijk}^{p} \otimes \tilde{x}_{ijk}^{p} = (4476,6029,7507,8996)_{LR}$$

$$z_3^* = \sum_{p=1}^{2}\sum_{i=1}^{3}\sum_{j=1}^{4}\sum_{k=1}^{2} \tilde{c}_{ijk}^{p} \otimes \tilde{x}_{ijk}^{p} = (12.9,21.6,35.5,50.2)_{LR}$$

The fuzzy optimal solution and total fuzzy transportation cost based on this proposed method have no negative part.

Now, we have formulated the following membership functions of model 2 is as follows:

$$\mu(z_1^1(x)) = \begin{cases} 0, & if Z_1^1(x) \geq U_1^1 \\ \dfrac{1649 - Z_1^1(x)}{1649 - 1313}, & if\ L_1^1 < Z_1^1(x) < U_1^1 \\ 1, & if Z_1^1(x) \leq L_1^1 \end{cases}, \quad \mu(z_1^2(x)) = \begin{cases} 0, & if Z_1^2(x) \geq U_1^2 \\ \dfrac{2232 - Z_1^2(x)}{2232 - 1781}, & if\ L_1^2 < Z_1^2(x) < U_1^2 \\ 1, & if Z_1^2(x) \leq L_1^2 \end{cases}, \quad \mu(z_1^3(x)) = \begin{cases} 0, & if Z_1^3(x) \geq U_1^3 \\ \dfrac{2881 - Z_1^3(x)}{2881 - 2253}, & if\ L_1^3 < Z_1^3(x) < U_1^3 \\ 1, & if Z_1^3(x) \leq L_1^3 \end{cases}$$

$$\mu(z_1^4(x)) = \begin{cases} 0, & if Z_1^4(x) \geq U_1^4 \\ \dfrac{3547 - Z_1^4(x)}{3547 - 2779}, & if\ L_1^4 < Z_1^4(x) < U_1^4 \\ 1, & if Z_1^4(x) \leq L_1^4 \end{cases}, \quad \mu(z_2^1(x)) = \begin{cases} 0, & if Z_2^1(x) \geq U_2^1 \\ \dfrac{4512 - Z_2^1(x)}{4512 - 4476}, & if\ L_2^1 < Z_2^1(x) < U_2^1 \\ 1, & if Z_2^1(x) \leq L_2^1 \end{cases}, \quad \mu(z_2^2(x)) = \begin{cases} 0, & if Z_2^2(x) \geq U_2^2 \\ \dfrac{6073 - Z_2^2(x)}{6073 - 6029}, & if\ L_2^2 < Z_2^2(x) < U_2^2 \\ 1, & if Z_2^2(x) \leq L_2^2 \end{cases}$$

$$\mu(z_2^3(x)) = \begin{cases} 0, & if Z_2^3(x) \geq U_2^3 \\ \dfrac{7561 - Z_2^3(x)}{7561 - 7507}, & if\ L_2^3 < Z_2^3(x) < U_2^3 \\ 1, & if Z_2^3(x) \leq L_2^3 \end{cases}, \quad \mu(z_2^4(x)) = \begin{cases} 0, & if Z_2^4(x) \geq U_2^4 \\ \dfrac{9060 - Z_2^4(x)}{9060 - 8996}, & if\ L_2^4 < Z_2^4(x) < U_2^4 \\ 1, & if Z_2^4(x) \leq L_2^4 \end{cases}, \quad \mu(z_3^1(x)) = \begin{cases} 0, & if Z_3^1(x) \geq U_3^1 \\ \dfrac{17.7 - Z_3^1(x)}{17.7 - 12.9}, & if\ L_3^1 < Z_3^1(x) < U_3^1 \\ 1, & if Z_3^1(x) \leq L_3^1 \end{cases}$$

$$\mu(z_3^2(x)) = \begin{cases} 0, & if Z_3^2(x) \geq U_3^2 \\ \dfrac{28.9 - Z_3^2(x)}{28.9 - 21.6}, & if\ L_3^2 < Z_3^2(x) < U_3^2 \\ 1, & if Z_3^2(x) \leq L_3^2 \end{cases}, \quad \mu(z_3^3(x)) = \begin{cases} 0, & if Z_3^3(x) \geq U_3^3 \\ \dfrac{44.2 - Z_3^3(x)}{44.2 - 35.5}, & if\ L_3^3 < Z_3^3(x) < U_3^3 \\ 1, & if Z_3^3(x) \leq L_3^3 \end{cases}, \quad \mu(z_3^4(x)) = \begin{cases} 0, & if Z_3^4(x) \geq U_3^4 \\ \dfrac{52.2 - Z_3^4(x)}{52.2 - 50.2}, & if\ L_3^4 < Z_3^4(x) < U_3^4 \\ 1, & if Z_3^4(x) \leq L_3^4 \end{cases}$$

The fuzzy optimal solution of model 2 using goal programming method is as follows:

$\tilde{x}_{1111}^* = (72, 72, 72, 87)_{LR}$	$\tilde{x}_{1211}^* = (128, 327, 327, 328)_{LR}$	$\tilde{x}_{1311}^* = (0, 0, 0, 0)_{LR}$	$\tilde{x}_{1411}^* = (0, 0, 0, 0)_{LR}$
$\tilde{x}_{2111}^* = (0, 0, 0, 1)_{LR}$	$\tilde{x}_{2211}^* = (0, 0,100, 183)_{LR}$	$\tilde{x}_{2311}^* = (0, 0, 0, 0)_{LR}$	$\tilde{x}_{2411}^* = (100, 100, 194, 210)_{LR}$
$\tilde{x}_{3111}^* = (628, 720, 720, 720)_{LR}$	$\tilde{x}_{3211}^* = (272, 272, 272, 288)_{LR}$	$\tilde{x}_{3311}^* = (0, 0, 0, 0)_{LR}$	$\tilde{x}_{3411}^* = (0, 0, 100, 100)_{LR}$
$\tilde{x}_{1122}^* = (600, 600, 600, 641)_{LR}$	$\tilde{x}_{1222}^* = (0, 0, 18, 18)_{LR}$	$\tilde{x}_{1322}^* = (0, 145, 145, 145)_{LR}$	$\tilde{x}_{1422}^* = (0, 0, 0, 0)_{LR}$
$\tilde{x}_{2122}^* = (0, 0, 81, 81)_{LR}$	$\tilde{x}_{2222}^* = (0, 0, 0, 0)_{LR}$	$\tilde{x}_{2322}^* = (200, 200, 200, 200)_{LR}$	$\tilde{x}_{2422}^* = (0, 45, 164, 164)_{LR}$
$\tilde{x}_{3122}^* = (0, 100, 100, 100)_{LR}$	$\tilde{x}_{3222}^* = (0, 0, 76, 76)_{LR}$	$\tilde{x}_{3322}^* = (0, 0, 0, 0)_{LR}$	$\tilde{x}_{3422}^* = (0, 1, 1, 92)_{LR}$
$\tilde{x}_{1121}^* = (0, 0, 0, 84)_{LR}$	$\tilde{x}_{1221}^* = (100, 100, 100, 100)_{LR}$	$\tilde{x}_{1321}^* = (300, 301, 401, 501)_{LR}$	$\tilde{x}_{1421}^* = (0, 0, 0, 0)_{LR}$
$\tilde{x}_{2121}^* = (0, 100, 100, 100)_{LR}$	$\tilde{x}_{2221}^* = (0, 1, 1, 1)_{LR}$	$\tilde{x}_{2321}^* = (0, 99, 99, 99)_{LR}$	$\tilde{x}_{2421}^* = (0, 100, 106, 106)_{LR}$
$\tilde{x}_{3121}^* = (0, 8, 108, 108)_{LR}$	$\tilde{x}_{3221}^* = (0, 0, 0, 0)_{LR}$	$\tilde{x}_{3321}^* = (0, 0, 0, 0)_{LR}$	$\tilde{x}_{3421}^* = (100, 100, 100, 284)_{LR}$
$\tilde{x}_{1112}^* = (0, 0, 0, 50)_{LR}$	$\tilde{x}_{1212}^* = (0, 1, 2, 2)_{LR}$	$\tilde{x}_{1312}^* = (100, 100, 100, 200)_{LR}$	$\tilde{x}_{1412}^* = (0, 54, 135, 144)_{LR}$

$\tilde{x}^*_{2112} = (0,\ 0,\ 0,\ 109)_{LR}$	$\tilde{x}^*_{2212} = (0,\ 0,\ 0,\ 91)_{LR}$	$\tilde{x}^*_{2312} = (0,\ 55,\ 55,\ 55)_{LR}$	$\tilde{x}^*_{2412} = (0,\ 0,\ 0,\ 0)_{LR}$
$\tilde{x}^*_{3112} = (0,\ 0,\ 19,\ 19)_{LR}$	$\tilde{x}^*_{3212} = (600,\ 699,\ 704,\ 713)_{LR}$	$\tilde{x}^*_{3312} = (0,\ 0,\ 100,\ 100)_{LR}$	$\tilde{x}^*_{3412} = (100,\ 100,\ 100,\ 100)_{LR}$

$l = (.828,\ .8284,\ .8952,\ .8952)_{LR}$, $d^-_1 = (0.828,\ 0.828,\ 0.828,\ .830)_{LR}$, $d^-_2 = (0.828,\ 0.828,\ 0.828,\ .888)_{LR}$, $d^-_3 = (0.6988,\ 0.8284,\ 0.8952,\ 0.8952)_{LR}$,

$z_1 = (1620, 2093.45, 2651.78, 3321.98)_{LR}$, $z_2 = (4483, 6049.01, 7526.66, 9018.54)_{LR}$, $z_3 = (17.632,.27.637, 42.452, 59.388)_{LR}$

Conclusion

For the first time Eco –friendly fully fuzzy multi objective multi-item solid transportation problem with LR-flat fuzzy numbers are formulated and solved. The main advantages of this proposed method is the optimal solutions arealso expressed in non-negative LR flat fuzzynumbers. Two models have been discussedin which one is non-eco-friendly and another one is eco-friendly. From the above result we observe that Emission cost of eco-friendly model is 65.69% lesser than the emission cost of non-eco-friendly model. Theprescribed methodology shows that the aspiration level of eco-friendlymodel is better than non-eco-friendly model. This model will be highly useful to the society as it has been focused to reduce the environment impact of transportation.

Reference

1. Chakraborty, and M. Chakraborty, cost – time minimization problem with fuzzy parameters: A case study Journal of transportation systems Engineering and Information Technology 10 (6) (2010) 53-63.
2. Ebrahimnejad, A new method for solving fuzzy transportation problem with LR flat fuzzy numbers, Information Sciences,(2016)
3. Ebrahimnejad, A simplified new approach for solving fuzzy transportation problems with generalized trapezoidal fuzzy numbers, Applied soft computing 19 (2014) 171-176.
4. Ebrahimnejad, A. duality approach for solving bounded linear programming with fuzzy variables based on ranking functions and its application in bounded transportation problems, International Journal of systems Science (2013), doi: 0.1080/00207721.2013.844285.
5. Ebrahimnejad, Sensitivity analysis in fuzzy number linear programming problems. Mathematical and Computer Modelling. 53(9-10) (2011) 1878-1888.
6. Ebrahimnejad, some new results in linear programs with trapezoidal fuzzy numbers: Finite convergence of the Ganesan and Veeramani's method and a fuzzy revised simplex method, Applied Mathematical Modelling, 35(9) (2011) 4526-4540.
7. D. Dubois, and H. Prade, Fuzzy sets and systems: theory and applications, Academic Press, New York (1980).
8. Development of suitable package for Transport of Guava.
9. DipankarChakraborty,Dipakkumarjana and TapankumarRoy,A new approach to solve multi-objective multi-choice multi-item Atanassov's intuitionistic fuzzy transportation problemusing chance operator,Journal of Intelligent and fuzzy systems 28 (2015)843-865.
10. E.D. Schell, "Distribution of a product by Several Properties", in proceedings of the 2nd symposium in Linear Programming, DCS/Comptroller, HQ US Air Force, pp.615-642, Washington, DC, USA, 1955.
11. F.L. Hitchcock, "The distribution of a product from several sources to numerous localities," Journal of Mathematics and Physics, Vol.20. pp.224-230, 1941.
12. FAQ: What are the advantages of Biodiesel over petroleum diesel – BIOCUBE™, html.
13. G. J. Klir, and B. Yuan, Fuzzy Sets and Fuzzy Logic, Theory and Applications, Prentice-Hall, PTR, New Jersey, (1995).
14. K. B. Haley, "The Solid transportation Problem", operations Research, Vol.10, pp.448-463, 1962.
15. Keith James ,Methodology for assessing the climate change impacts of packaging optimization under the courtauld commitment phase 2,Wrap,Dec 2010.
16. M.N.Amin,M.A.Hossain,M.S.Miah,M.S.Hassan and M.A.Hoque,Development of suitable package for transportation Guaua,Bangladesh J.Agril.Res.39(2):337-350,June 2014.

17. P.Kundu,S.Kar,M.Maity,Multi objective multi-item solid transportation problem in fuzzy environment,Applied mathematical modeling,2013.
18. Report on Packaging of Fresh Fruits And Vegetables for Exports.
19. S.Pramanik,D.K.Jana,M.Maity,Multi objective solid transportation problem in imprecise environment ,Journal of transportation security,2013.
20. www.Gaj supply.com industrial products.
21. www.Transport-Logistics (1) Packaging Cost. pdf.
22. www.wrap.org.uk.
23. YaunChen,KanyuChen,YangLI,Simulation on Influence Mechanism of Environmentalfactors to producers food security Behaviour in supply chain,Eighth international conference on Fuzzy systems and Knowledge Discovery(2011).
24. Zadeh L.A.,1965,Fuzzy sets, Information and control,8,338-353.