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Statement & Understanding of Rolle's Theorem

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Abstract:-The purpose of this paper is to understand Rolle's in every perspective basically theoretically and graphically. The knowledge component required for the better understanding of this theorem involve limit continuity and derivability. The knowledge of this theorem is compulsory in application of many theorems on advanced calculus like Cauchy Mean Value Theorem Taylor's theorem. This theorem is study as special case of mean value theorem in differential calculus.

The beauty of this theorem reflects as its connection with real life. A ball, when thrown up come down and during the movement, it changes its direction at some point to come down. It show that velocity of the ball which is thrown upwards must be zero at a point.

History of Rolle's Theorem:-Indian mathematician Bhaskara-II created the knowledge of Rolle 's theorem in 1114-1185 but in 1691, Michel Rolle's created the knowledge regarding polynomial functions he did not use the concept of differential calculus. This theorem was first proved by Cauchy in 1823 as a corollary of Mean Value Theorem the name "Roll's Theorem" was first used by Moritz Wilhelm Drobisch of Germany in 1834.

Introduction and Statement:-Rolle's Theorem state that any real valued differentiable function that attains equal value at two distinct points must have at least one point between end points at that point the first derivative is zero or slope of tangent line at this point is parallel to X-axis

3. ROLLE'S THEOREM:-If f be a real valued function with domain [a,b] satisfying these three conditions.

(i) f is continuous in the closed interval [a,b]
(ii) f is derivable in the open interval (a,b)
(iii) $f(a) = f(b)$

then there exists $c \in (a, b)$ s.t. $f'(c) = 0$

Proof: since f is continuous function over a closed interval so bounded and attains its bounds. If M and m be its least upper bound and greatest lower bound of f on [a, b], then there exist c, d ∈ [a, b] such that

$$f(c) = M \text{ and } f(d) = m$$

If m =M (Trivial Case)

Then f (x) = M = m for all x ∈[a,b]

i.e., f (x) is a constant function.

=> $f'(x) = 0$ for all x∈ [a , b]

=> $f'(c) = 0$ for all c∈[a, b]

Now, we take the case when m≠ M (General Case)

As f (a) = f(b)

Thus at least one of m and M is different from f(a) and f(b)

Let M be different from each of f(a) and f(b)

c∈(a, b) s.t. M = f(c) ≠f(a) and M = f(c) ≠f(b)

Since f is derivable in (a, b) and c∈(a, b), so f is derivable at c

=> L f'(c)=R f(c) ......(1)

Now, $$Lf'(c) = \lim_{h\to 0+} \frac{f(c-h)-f(c)}{-h} = \lim_{h\to 0+} \frac{f(c+h)-f(c)}{h}$$

Since f (c) = M =l.u.b. of f on [a,b] => f (c-h) ≤ f(c) => f (c-h) - f (c) ≤ 0

This Implies that, L f'(c) ≤ 0 ……(2)

Also, $$R\,f'(c) = \lim_{h\to 0+} \frac{f(c+h)-f(c)}{h}$$

Since f(c) = M = lub of f on [a, b] => f(c + h)≤ f(c) => f(c + h) - f(c) ≤ 0

This implies that R f'(c) ≤0 ……(3)

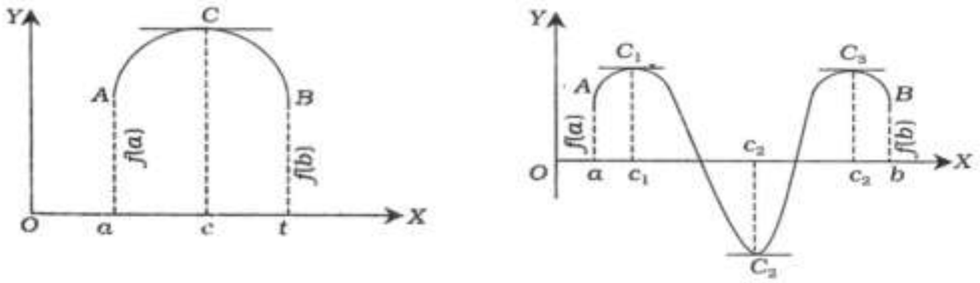
From (1), (2) and (3), we conclude that Lf'(c) = Rf'(c) = 0 => f'(c) = 0

Similarly we can prove the result when m = f(d) is different from f(a) and f(b).

3.1 Geometrical Interpretation of Rolle's theorem :- Let A = f(a) and B = f(b) so that AB represents the graph of f in [a, b] .

If f(x) is continuous on [a, b] , then the graph of f(x) is without breaks.

If f(x) is derivable on (a, b), then the graph of f(x) has a tangent at every point between A and B.



If f(a) = f(b), then A and B will lie on the same horizontal level.

The graph satisfying above conditions is shown in the figures. The careful observation of these figures implies that there exists at least one point C on the curve between A and B where the tangent is parallel to x-axis.

i.e. at C, $f'(x) = 0$.

i.e., there exists some c between a and b such that $f'(c) = 0$.

There must exist another form of Rolle's Theorem also, we can study that form in similar way.

Application of Rolles. Theorem in the motion of a ball:-The position of a ball at a point when it is thrown upward from height of 3 feet with the initial velocity 5 feet/sec $f(f) = -15+^2 +45t+3$ find the time at maximum height the track of the ball without break So f(t) is a continuous function and ball takes 1 to 2 second from initial post point to terminal point.

1. So f(f) is a continuous function on [1,2]

2. Tangent at every point of the graph of f (f) is unique so it is derivable on exit soft is derivable on (1,2).

3. Value of f (f) is same at initial point and terminal point in upward and downward positions so $f(1) = f(2)$

So ∃ at least one $t_1 \in (1,2)$ s.t

$f'(t_1) = .0$ $\qquad$ $-30\, t_1 + 45 = 0$

$$t_1 = \frac{45}{30} = 1.5$$

at this time velocity is zero

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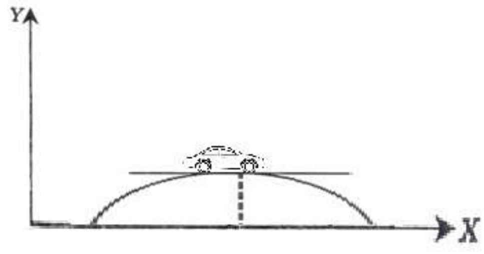
Application of Rolles theorem in Driving a Car on a flyover:-While driving a Car/Motor on a flyover, Rolles theorem is applicable as the flyover is without break. So it is Continuous function between initial point and terminal point. We can draw unique tangent at every point between initial and terminal point on its graph, value at initial point and terminal points are same , so ∃ one point between initial and terminal point at which tangent is along horizontal direction. It means at this point the car is moving along horizontal direction.



Conclusion:-If all the three conditions of this theorem satisfied, it confirms the existence of at least one point where tangent is along horizontal direction. It help in study of nature of functions. It is very needful in study of advanced calculus. We cannot study some basic theorem like Cauchy's Mean Value Theorem and taylors theorem without basic knowledge of Rolle's Theorem.

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