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# DIRECTED GRAPHS : RANKING THE PARTICIPANTS IN A TOURNAMENT

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**Definition :-** A directed graph is graph, i.e. a set of objects (called vertices or nodes) that are connected together, where all the edges are directed from one vertex to another. In contrast a graph where the edges are bidirectional is called an undirected graph.

## ABSTRACT :-

**RANKING THE PARTICIPANTS IN A TOURNAMENT**

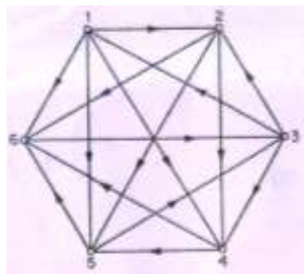
A number of players each play one another in a tennis tournament. Given the outcomes of the games, how should the participants be ranked ?

Consider, for example, the tournament of figure 1. This represents the result of a tournament between six players ; we see that player 1 beat players 2, 4,5 and 6 and lost to player 3, and so on.

One possible approach to ranking the participants would be to find a directed Hamilton path in the tournament and then rank according to the position on the path. For instance, the directed Hamilton path (3, 1, 2, 4,5, 6) would declare player 3 the winner, player 1 runner - up and so on. This method of ranking, however, does not bear further examination, since a tournament generally has many directed Hamilton paths ; our example has (1, 2, 4, 5, 6, 3), (1, 4, 6, 3, 2, 5) and several others.

Another approach would be to compute the scores (numbers of games won by each player) and compare them. If we do this we obtain the score vector.

$S_1 = (4,3,3,2,2,1)$




The drawback here is that this score vector does not distinguish between players 2 and 3 even though players 3 beat players with higher scores than did player 2. We are thus led to the second - level score vector.

$$S_2 = (8, 5, 9,3,4,3)$$

in which each player's second - level score is the sum of the scores of the players he heat. Player 3 now ranks first. Continuing this procedure we obtain further vectors.

$S_3 = (15, 10, 16, 7, 12, 9)$

$S_4 = (38, 28, 32, 21, 25, 16)$

$S_5 = (90, 62,87, 41, 48, 32)$

$S_6 = (183, 121, 193, 80, 119, 87)$

The ranking of the players is seen to fluctuate a little, player 3 vying with players 1 for first place. We shall show that this procedure always converges to a fixed ranking when the tournament in question is disconnected and has at least four vertices. This will then lead to a method of ranking the players in any tournament.

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In a diconnected digraph D, the length of a shortest directed (u, v) - path is denoted by $d_D$ (u, v) and is called the distance from u to v; the directed diameter of D is the maximum distance from any one vertex of D to any other.

Theorem :- Let D be a diconnected tournament with $v \geq 5$, let A be the adjacency matrix of D. Then $A^{d+3} > O$ (every entry positive), where d is the directed diameter of D.

Proof : The (i, j)th entry of $A^k$ is precisely the number of directed ($v_i$ $v_j$) - walks of length K in D. We must therefore show that, for any two vertices $v_i$, and $v_j$ (possibly identical), there is a directed ($v_i$, $v_j$) - walk of length d + 3.

Let $d_{ij} = d(v_i\ v_j)$. Then $0 \leq d_{ij} \leq d \leq v - 1$ and therefore

$$3 \leq d - d_{ij}+3 \leq v + 2$$

If $d - d_{ij} + 3 \leq v$ then, there is a directed $(d - d_{ij} + 3)$ - cycle C containing $v_j$ . A directed ($v_i$, $v_j$) - path p of length $d_{ij}$ followed by the directed cycle C together form a directed ($v_{i,}v_j$) - walk of length d + 3, as desired.

There are two special cases. If $d - d_{ij} + 3 = v + 1$, then P followed by a directed (v - 2) - cycle through $v_j$ followed by a directed 3- cycle through $v_j$ constitute a directed ($v_{i,}v_j$) - walk of length d + 3 (the (v - 2) - cycle exists since $v \geq 5$) ; and if $d - d_{ij} +3 = v + 2$, then P followed by a directed (v - 1) - cycle through $v_j$ followed by a directed 3-cycle through $v_j$ constitute such a walk.

A real matrix R is called primitive if $R^k > 0$ for some K.

Corollary. The adjacency matrix A of a tournament D is primitive if and only if D is disconnected and $v \geq 4$.

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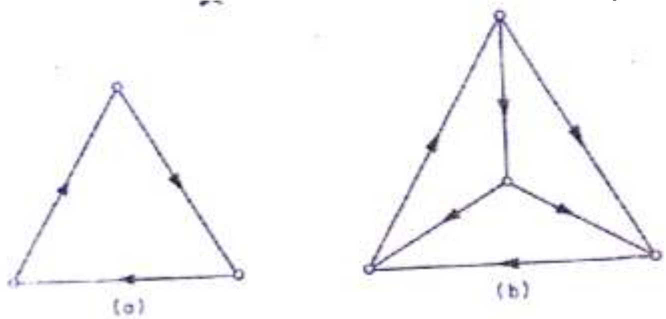
Proof If D is not diconnected, then there are vertices $v_i$ and $v_j$ in D such that $v_j$ is not reachable from $v_i$. Thus there is no directed $(v_i, v_j)$ - walk in D. It follows that the (i, j) th entry of




v ≥ 5 then, by theorem $A^{d+3} > 0$ and so A

ıt on three vertices (figure 2a), and just one

). It is readily checked that the adjacency

matrix of the 3- vertex tournament is not primitive, and it can be shown that the ninth power of the adjacency matrix of the 4- vertex tournament has all entries positive.

Returning now to the score vectors, we see that the ith - level score vector in a tournament D is given by

$$S_i = A^i J$$

where A is the adjacency matrix of D, and J is a column vector of 1's. If the matrix A is primitive then, the eigenvalue of A with largest absolute value is a real positive number r and, furthermore,

$$\lim_{i \to \infty} \left(\frac{A}{r}\right)^i j = s$$

Where s is a positive eigenvector of A corresponding to r. Therefore, by corollary, if D is a diconnected tournament on at least four vertices, the normalised vector $\underset{s}{\rightarrow}$(with entries summing to one) can be taken as the vector of relative strengths of the players in D. In the example of figure - 1, we find that (approximately)

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$$r = 2.232 \text{ and } \underset{s}{\rightarrow} = (.238, .164, .231, .113, .150, .104)$$

Thus the ranking of the players given by this method is 1,3, 2, 5, 4, 6.

If the tournament is not diconnected, then its dicomponents can be linearly ordered so that the ordering preserves dominance. The participants in a round - robin tournament can now be ranked according to the following procedure.

Step 1 In each dicomponents on four or more vertices, rank the players using the eigenvector $\underset{s}{\rightarrow}$; is a dicomponent on three vertices rank all three players equal.

Step 2 Rank the dicomponents in their dominance - preserving linear order $D_1$, $D_2$.... $D_m$; that is, if i < j then every arc with one end in $D_i$, and one end in $D_j$ has its head in $D_j$.

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