TAX EVASION AND CHOICE OF FORMAL AND INFORMAL SECTORS

Dr. Dibyendu Banerjee*

Abstract
The paper considers imposition of unit tax by the government on a firm selling product in a monopoly market. The firm has the option either to pay tax or to evade it. Tax evasion can be done either by selling the product in the formal sector or in the informal sector. The firm can evade the tax in the formal sector by under reporting its true quantity. On the other hand, it can evade by producing in the informal sector by adopting some illegal means of production. In this framework, it is observed that the firm pays tax only when the chance of getting detected is high. Interestingly, the paper highlights that imposition of a specific unit tax might not reduce tax evasion. If there is an increase in the efficiency of government monitoring and enforcement of penalties, tax evasion might be reduced significantly.

Key words: Tax Evasion, Monopoly, Informal Sector, Formal Sector.

*Assistant Professor, Dept. of Economics, Serampore College, Hooghly
1. INTRODUCTION
An economy or economic system consists of the production, distribution or trade, and consumption of limited goods and services by different agents in a given geographical location. It includes the combination of several institutions, entities, agencies, decision-making processes and patterns of consumption that make up the economic structure of a specific community. This is a type of social system which defines how all the entities in an economy interact. Government is a very important part of any economy and has various important functions to perform. The role of government arises from the fact that the market often fails to allocate resources efficiently. It sometimes fails to distribute and stabilize the economy in the optimal manner. Government has the responsibility to provide social infrastructure (like, defense, education, health services) to the citizens and impose tax to discourage the conspicuous consumption. When the economy is in any unstable position, like experiencing high rates of inflation, unemployment, trade deficit etc. it is only the government which can undertake some correctional measures.

One common practice to raise revenue is to impose taxes. Direct taxes are those which are imposed initially on the individual or the household that are meant to bear the burden. While, indirect taxes are taxes which are imposed at some other point in the system but are shifted to whomever is supposed to be final bearer of the burden. Personal taxes, such as the individual income tax, wealth tax, gift tax etc. are thus ‘direct’ and commodity taxes such as sales tax, excise tax etc. are ‘indirect’. On grounds of equity indirect taxes are inferior to well-designed personal taxes imposed directly on a particular tax payer’s ability to pay. In developing countries (including India), the share of indirect tax in total tax revenue remains high most of the time. The reason is that, in a developing country the income level remains low, the narrow tax base does not cover all economic agents and an individual evades tax by under-reporting his true income. So the government relies heavily on indirect taxes as a source of revenue. But a seller can evade tax in two ways- either by producing in the formal sector or by producing in the informal sector.

On the basis of employment conditions, there are two types of sectors in an economy: formal and informal. A formal sector of an economy is a well-defined sector or an organized sector that contributes substantially to the gross national product (GNP) and gross domestic product (GDP) of the country. It is within the purview of the government monitoring. Alternatively, sector which encompasses all jobs with normal hours and regular wages, and are recognized as income
sources on which income taxes must be paid is known as formal sector. On the other hand, there is a vast informal sector which is often described as parallel economy or unorganized sector. In the 1950s and the 1960s empirical investigation widely observed a dichotomy in the urban economies, where besides the organized industrial sector, there exist an unorganized, unprotected, traditional sector. The notion of ‘informal sector’ was given by W. Arthur Lewis (1954). But, he did not attribute the term as “informal sector”. British anthropologist Keith Hart (1971) first identified ‘informal sector’ and introduced it as part of the urban labour force that takes place outside the formal labour market. A seminal ILO Report (1972) on Kenya introduces the following characteristics of the informal sector: easy entry for the new enterprises, reliance on indigenous resources, family ownership of enterprises, small scale of operations and low productivity, labour-intensive and adapted technology, reliance of workers on informal sources of education and skills, skills acquired outside the formal school system and the bypassing of regulations, unregulated and competitive markets, and, lack of governmental support. Though it should be pointed out that all the informal activities do not always involve illegal activities rather it is a product of rational behaviour of entrepreneurs that desire to escape state regulations.

Present paper models a monopoly market where a monopolist selling a product and charging a price above its marginal cost and the government imposes a per unit tax on the goods sold by it. This tax is used to finance some of the developmental expenditures of the economy. In such situation, the firm has the option to pay the tax or evade it. It can evade the tax either by under-reporting its quantity produced and sold in the formal sector or by producing in the informal sector. The firm would choose the more profitable option. Now, there is a risk involved in under-reporting the true quantity sold - being caught and penalized by the tax officials. When the firm is detected, the seller in the formal sector has to pay the remaining amount of tax and along with that it has to pay a penalty and the firm loses its business in the informal sector. The chance of detection is positively related with the efficiency of the government administration. To keep the calculation simple this theoretical analysis does not consider the instance of bribing the government officials by the firms to escape penalty. The analysis leads to some conditions for which the firm pays the full tax or it evades the tax by producing in the formal or in the informal sector. The rest of the paper is organized as follows. The next section presents the literature survey followed by data and methodology. Section four describes the structure of the model the final section concludes the paper.
2. LITERATURE SURVEY

The fact that the share of indirect taxes always remains quite high in total tax revenue particularly in a developing country is evident from the Indian experience found in Rakshit (2009). In order to reduce the burden of indirect tax, there have been efforts from the Indian planners to reform the tax system to increase direct tax compliance. Still indirect taxation plays a dominating role in raising tax revenue by the government [Bagchi (2005)].

Lewis (1954) first gave the concept of ‘informal sector’ in his celebrated paper *Economic Development with Unlimited Supplies of Labour* but the term ‘Informal Sector’ is originally attributed to the economic development model put forward by Hart (1973) in his empirical study “*Informal Income Opportunities and Urban Employment in Ghana*”.

Laffont and Tirole (1986) and Lewis and Sappington (1988) focused on the design of an optimal regulation contract in a monopoly under asymmetric information. On the first paper the regulator was not aware of the monopolist’s marginal costs, while in the second paper private information was related to the market’s demand function. They all concluded that the regulator has to pay a price above the marginal cost to avoid an untruthful report by the monopolist. Laffont and Martimort (1997) and Tangeras (2002) introduced the possibility of collusion between two firms in a context of complete regulation under asymmetric information.

The objective of my study is to find out whether a firm pays the tax or not? Again if the firm decides to evade the tax, will it choose the formal sector or the informal sector? Thus, it is interesting to see that how the answers to these questions are addressed.

3. DATA AND METHODOLOGY

Since my paper is a theoretical, no primary or secondary data is used here. Rather I have tried to show my result by building up a theoretical model. The analysis considers a model concerning a monopoly market which enjoys certain monopoly power, where the government imposes a per unit tax on the quantity sold to finance the developmental activities of the economy. In this situation a firm can either pay the tax or evade it. If the firm decides to evade the tax it can evade the tax by producing in the formal or in the informal sector.
Following methodologies are followed in this paper:

I. Profit levels in each case (when the firm pays the full tax or when the firm evades the tax either in the formal or in the informal sector) are calculated algebraically.

II. The firm in the formal sector evades the tax by under-reporting its quantity produced and sold and there is a chance of being detected and then the firm has to pay the entire tax and a penalty amount. The chance of detection and non-detection is calculated with the help of probability both in formal and informal sectors.

III. In the informal sector, when the firm is detected then it loses its business as it undertakes illegal means of production.

IV. To keep the algebra simple any possibility of corruption, rent-seeking activities, bribery is not considered.

4. ANALYSIS

An economy or economic system consists of the production, distribution or trade and consumption of limited goods and services by different agents in a given geographical location. Suppose a firm is producing a single good having some monopoly power i.e. it is able to charge a price above its marginal cost. i.e., \( P > MC \). The firm faces a linear demand function for its product i.e. \( p = a - bq \) where \( a, b > 0 \). The average cost (AC) and marginal cost (MC) of the firm are identical. Marginal cost, \( c \) is constant where \( c > 0 \). No fixed cost is assumed here. For simplicity we do not consider the instance of bribing the government officials by the firms to escape. In such situation the government imposes a unit tax on the quantity produced and sold by the firm. Let the tax rate be fixed at Rs. \( t \) per unit of quantity sold. We assume that the government imposes such tax only to finance some development expenditure. In this case the firm has two options- either it can choose to pay the tax or, it can choose to evade it. Definitely, the firm would choose that option which offers more profit. The following section describes the conditions under which a particular option becomes equilibrium decision of the firm. Let us consider the first case when, the firms pays the tax.


Case I: The firm pays the full tax to the government

Suppose the firm produces q units of output after tax. Thus, it has to pay a tax of amount Rs. tq to the government. So the net cost to the firm becomes \((cq + tq)\). The firm earns a revenue of the amount pq by selling its output in the market. So the post-tax profit function of the firm becomes:

\[
\pi = pq - cq - tq
\]  

(1)

Substituting \(p = a - bq\) in equation (1) the profit function can be rewritten as:

\[
\pi = aq - bq^2 - cq - tq
\]  

(2)

Firm maximizes equation (2) subject to \(q > 0\) and obtains the following first order condition:

\[
(a-c-t) - 2bq = 0
\]

or, \(q = \frac{a-c-t}{2b}\)

The second order condition for profit maximization is also satisfied as \(b > 0\); \(\frac{d^2\pi}{dq^2} = -2b < 0\)

Putting this value of q in equation (2) the equilibrium value of profit of the firm can be obtained. Let us call that value as \(\pi^*\). Thus,

\[
\pi^* = \frac{(a-c-t)^2}{2b}
\]

(3)

Here, optimum profit, when the firm is paying full tax, is denoted by \(\pi^*\).

Now, we will consider the case when the firm chooses to evade the tax. A firm can evade the tax in two ways:

- It can evade the tax by producing in the formal sector, or by producing in the informal sector.
- The firm would choose that option which offers more profit. Let us consider the cases one by one.

Case-II: Tax Evasion in Formal Sector

The formal sector offers job security and employment benefits and covers those enterprises of work where the terms of employment are regular and people have assured employment. Suppose, the firm attempts to evade a part of the tax liability by under-reporting the quantity it sells to the government. For example, people generally buy medicines of small quantity and do
not bother about the bills. Thus the medicine seller reveals only the billed amount as quantity sold to the government.

We assume the firm produces and sells an amount q but reveals only $\theta$ fraction of q as the quantity sold; $\theta \in (0, 1)$. So the firm pays an amount $(t\theta q)$ as tax to the government instead of paying tq amount. Therefore, it enjoys a profit of the amount $(pq - cq - t\theta q)$. But it operates under the risk or threat of being caught by the tax officials while evading tax. Let the probability of not being detected be $\rho$. Thus, the firm gets profit amount $(pq - cq - t\theta q)$ with probability $\rho$.

But what happens if the firm is detected? When the firm is detected while evading tax it has to pay the remaining amount of tax i.e. $(1 - \theta)tq$ and along with that it pays a fine. Let the rate of penalty by $r\%$ of the amount evaded. Thus the amount paid as fine becomes $r(1 - \theta)tq$. In this case the profit of the firm amounts to $[pq - cq - t\theta q - (1 + r)(1 - \theta)tq]$. This amount is realized with probability $(1 - \rho)$. As it is the probability of being detected by the government. The expected profit function of the firm becomes:

$$\pi^E = \rho(pq - cq - t\theta q) + (1 - \rho)\left[pq - cq - t\theta q - (1 + r)(1 - \theta)tq\right]$$

$$= \rho(aq - bq^2 - cq - t\theta q) + (1 - \rho)\left[aq - bq^2 - t\theta q - (1 + r)(1 - \theta)tq\right]$$

(4)

The firm maximizes equation (4) subject to $q > 0$ and obtains the following first order condition:

$$a - c - t\left[(1 - \rho)\left[1 + r(1 - \theta)\right] + \rho \theta\right] - 2bq = 0$$

or, $q = \frac{a - c - t\left[(1 - \rho)\left[1 + r(1 - \theta)\right] + \rho \theta\right]}{2b}$

The second order condition for profit maximization requires $\frac{d^2\pi^E}{dq^2} = -2b < 0$.

Substituting this value of q in equation (4) we get the equilibrium value of expected profit as:

$$\pi^{E*} = \frac{\left[a - c - t\left[(1 - \rho)\left[1 + r(1 - \theta)\right] + \rho \theta\right]\right]^2}{4b}$$

(5)

Here, optimum profit in the formal sector is $\pi^{E*}$

---

1 See Appendix A for details derivation.
Now, let us consider the case when the firm decides to evade the tax by choosing the informal sector for production.

**Case-III: Tax Evasion in Informal Sector**

Informal sector is marked by low wages and lack of job security. The employment condition in this sector is very much different from the formal ones. It is that part of an economy that is not taxed, monitored by any form of government, or included in any Gross National Product (GNP), unlike the formal sector.

Suppose the firm considers the alternative way to produce and sell in the informal sector to avoid tax. In such case the cost of production would be smaller than that in the formal sector. The reason is the firm employs labour at a wage rate which is below the standard set by the government. In this case the bargaining power resides with the firm as the workers are unorganized. This fact is reflected in our assumption that the marginal cost of producing in informal sector is $\tilde{c} < c$. Further, the demand faced by a firm in informal sector is different from that in formal sector. The highest price that a buyer is willing to pay for a particular unit is much lower in informal sector. For example, people would be willing to pay much more to buy a T-shirt from Adidas or Nike but the same person would be willing to pay much less for a T-shirt sold on pavement. Always this may not be the case. In line with this argument let the demand function faced by the firm is $p = \tilde{a} - bq$, where $\tilde{a} < a$.

Producing and selling in the informal sector is not always illegal. But in my project it is assumed that the firm operates by using some illegal means viz. encroaching public property like roadsides, footpaths etc. throw wastes into the drainage which blocks normal water flow in it or hooking electricity and so on. This attracts penalty when caught. So, let $\alpha \in (0,1)$ be the probability of being detected for producing in informal sector. In that case the firm loses its business and earns nothing. On the other hand, if it is not detected then it earns a profit of amount $(pq - \tilde{c}q)$ with probability $(1 - \alpha)$.

Thus the expected profit function of the firm in such situation becomes:
The firm maximizes the above function subject to $q > 0$ and obtains the following first order condition:

$$(1-\alpha)(\bar{a} - 2bq - \bar{c}) = 0$$

Since, $\alpha \in (0,1)$ so we have $q = \frac{\bar{a} - \bar{c}}{2b}$.

The second order condition for profit maximization requires

$$\frac{d^2 \pi}{dq^2} = -2(1-\alpha)b < 0,$$

which is automatically satisfied.

Substituting this value of $q$ in equation (6) we obtain the equilibrium value of expected profit as:

$$\bar{\pi} = \frac{(1-\alpha)(\bar{a} - \bar{c})^2}{4b}$$

Here, optimum profit when the firm is evading the tax by producing in the informal sector is denoted by $\bar{\pi}^*$.

**Results:**

I. Comparing equations (3) and (5), evading tax in the formal sector is preferred to paying full tax if $\pi^{E*} > \pi^*$. It requires:

$$\left[ a - c - t\left[ (1-\rho)\left[ 1+r(1-\theta) \right] + \rho \theta \right] \right]^2 \times \frac{(a-c-t)^2}{4b} \times \frac{(a-c-t)^2}{4b}$$

or, $\rho > \frac{r}{1+r}$

Thus, between paying full tax and evading tax by producing in the formal sector, the former will be an equilibrium outcome if $\rho \in \left( 0, \frac{r}{1+r} \right)$ and the latter will be preferred when $\rho \in \left( \frac{r}{1+r}, 1 \right)$.

---

See Appendix B for details derivation.
II. Comparing equations (5) and (7), evading tax by producing in the formal sector is preferred to the same in informal sector if $\pi^e > \pi^i$. So we have the following:

$$\frac{a - c - t\left[(1 - \rho)(1 + r(1 - \theta)) + \rho\theta]\right]}{4b} \left(1 - \alpha\right) \left(a - c\right)^2$$

or, $\alpha > 1 - \left[\frac{a - c - t\left[(1 - \rho)(1 + r(1 - \theta)) + \rho\theta]\right]}{a - c}\right]^2$ (9)$^3$

$$\alpha' = 1 - \left[\frac{a - c - t\left[(1 - \rho)(1 + r(1 - \theta)) + \rho\theta]\right]}{a - c}\right]^2$$

Let us call

Thus, between evading tax in formal sector and informal sector the former option will be preferred if $\alpha \in (\alpha', 1)$; otherwise for $\alpha \in (0, \alpha')$ the latter option will be preferred.

III. Comparing equations (3) and (7) we can say that evading tax in the informal sector is preferred to paying the full tax when $\pi^i > \pi^*$ i.e.

$$\frac{(1 - \alpha) \left(a - c\right)^2}{4b} > \frac{(a - c - t)^2}{4b}$$

or, $\alpha < 1 - \left(\frac{a - c - t}{a - c}\right)^2$ (10)$^4$

We call $\alpha'' = 1 - \left(\frac{a - c - t}{a - c}\right)^2$.

Thus evading tax by producing in the informal sector is more profitable than paying tax when $\alpha \in (0, \alpha'')$. Otherwise, for $\alpha \in (\alpha'', 1)$ paying tax is more profitable. These results can be summarized in terms of the following proposition.

**Proposition: I**

---

$^3$ See Appendix C for details derivation.

$^4$ See Appendix D for details derivation.
I. In a situation where the government imposes unit tax on the producers, the firm can either pay the tax or it can evade the tax by producing in the formal or informal sector. The firm pays the full tax only when \( \rho \in \left( 0, \frac{r}{1+r} \right) \) and \( \alpha \in (\alpha'', 1) \).

II. It evades tax by selling in the formal sector only when \( \rho \in \left( \frac{r}{1+r}, 1 \right) \) and \( \alpha \in (\alpha', 1) \).

III. It can also evade tax by selling in the informal sector only when \( \alpha \in (0, \alpha') \) and \( \alpha \in (0, \alpha'') \), i.e. \( \alpha \in (0, \alpha') \cap (0, \alpha'') \).

5. CONCLUSION:-
This theoretical paper models a monopoly market where the government imposes a per unit tax on the good sold by it. The tax is imposed solely with the revenue rising motive to finance any development expenditure or to raise the welfare of the economy. In such a situation, the firm has two options either it can pay the tax or it can evade it. The firm can evade the tax by producing in the formal or in the informal sector. Always, a risk is involved in evading the tax and always there is a chance of being caught by the tax officials. In the formal sector the firm can evade the tax by under-reporting the true quantity sold. On the other hand, the firm can adopt some illegal means and can produce in the informal sector. Also there is a chance of getting detected in the informal sector. When detected, in the formal sector the firm has to pay some penalty for under-reporting the true quantity sold and in the informal sector, the firm loses its business for adopting illegal means of production. In such framework, the conditions for which a particular outcome is preferred by the firm are obtained.

It is observed that the firm pays the tax only when the chance of getting detected is high. Otherwise the firm might choose to escape taxation either by producing in the formal sector or in the informal sector. The chance of detection increases with the efficiency of the government administration. In developed countries the government administration is highly efficient and its monitoring is also very stringent. So the incidence of tax evasion is very little there and the size of informal sector is also very small. On the other hand, in the developing countries the
government administration is relatively weak which allows widespread tax evasion in the formal sector and also expansion of the informal sector.

The result has an interesting policy implication. Often the tax reforms measures suggest widening of tax base, increasing the tax net and so on to increase tax compliance. But it overlooks an important perspective that apart from such measures it is also necessary to increase the efficient Government administration to ensure higher tax compliance. This study considers a linear demand function and constant marginal cost to keep the algebraic calculation simple. This is a limitation of the study. To keep the algebra simple we do not consider the instance of bribing the government officials by the firms to escape penalty. Both of this could well be the future agenda of research.

**APPENDICES:**

**Appendix A: [Derivation of equation 5]**

\[
\pi^E = \rho \left( pq - cq - t\theta q \right) + \left(1 - \rho\right) \left[ pq - cq - t\theta q - \left(1 + r\right)(1 - \theta)q \right] \\
= \rho \left( aq - bq^2 - cq - t\theta q \right) + \left(1 - \rho\right) \left[ aq - bq^2 - t\theta q - \left(1 + r\right)(1 - \theta)q \right]
\]

Differentiating both sides with respect to \( q \), we get,

\[
\frac{d\pi^E}{dq} = \rho \left( a - 2bq - c - t\theta \right) + \left(1 - \rho\right) \left[ a - 2bq - c - t\theta - \left(1 + r\right)(1 - \theta)q \right] = 0
\]

\[
\Rightarrow \rho \left( a - 2bq - c - t\theta \right) + \left[ a - 2bq - c - t\theta - \left(1 + r\right)(1 - \theta)q \right] = 0
\]

\[
\Rightarrow a - c - t \left[ \theta + (1 + r)(1 - \theta) \right] + \rho \left(1 + r\right)(1 - \theta)q = 2bq
\]

\[
2bq = a - c - t \left[ \left(1 - \rho\right)(1 + r(1 - \theta)) + \rho\theta \right]
\]

\[
\Rightarrow q = \frac{a - c - t \left[ \left(1 - \rho\right)(1 + r(1 - \theta)) + \rho\theta \right]}{2b}
\]

The second order condition requires \( \frac{d^2\pi^E}{dq^2} < 0 \),

Here, \( \frac{d^2\pi^E}{dq^2} = -2b < 0 \)

Now, putting the value of \( q \) in equation (4), we get,
\[
\pi^{e^*} = \rho (a - bq - c - t\theta) q + (1 - \rho)[a - bq - c - t\theta - (1 + r)(1 - \theta)t] q
\]

\[
\Rightarrow \pi^{e^*} = \rho \left[ a - b \left( \frac{a - c - t(1 - \rho)(1 + r(1 - \theta)) + \rho \theta}{2b} \right) - c - t\theta \right] \left( \frac{a - c - t(1 - \rho)(1 + r(1 - \theta)) + \rho \theta}{2b} \right)
\]

\[
+ (1 - \rho) \left[ a - b \left( \frac{a - c - t(1 - \rho)(1 + r(1 - \theta)) + \rho \theta}{2b} \right) - c - t\theta - (1 + r)(1 - \theta)t \right] * \left[ \frac{a - c - t(1 - \rho)(1 + r(1 - \theta)) + \rho \theta}{2b} \right]
\]

\[
\Rightarrow \pi^{e^*} = \left[ \frac{a - c - t(1 - \rho)(1 + r(1 - \theta)) + \rho \theta}{2b} \right]^2
\]

\[
(5)
\]

Appendix B: [Derivation of condition 8]

\[
\left[ \frac{a - c - t(1 - \rho)(1 + r(1 - \theta)) + \rho \theta}{2b} \right]^2 \left( \frac{(a - c - t)^2}{4b} \right)
\]

\[
\Rightarrow \left[ \frac{a - c - t(1 - \rho)(1 + r(1 - \theta)) + \rho \theta}{2b} \right]^2 \left( a - c - t \right)
\]

\[
\Rightarrow \left[ \frac{a - c - t(1 - \rho)(1 + r(1 - \theta)) + \rho \theta}{2b} \right] \left( a - c - t \right)
\]

\[
\Rightarrow \left[ \frac{(1 - \rho)(1 + r(1 - \theta)) + \rho \theta}{2b} - 1 \right] 0
\]

\[
\Rightarrow [(1 - \rho)(1 + r(1 - \theta)) + \rho \theta] 1
\]
\[\Rightarrow [(1 - \rho)(1 + r - r\theta + \rho\theta)]1\]
\[\Rightarrow 1 + r - r\theta - \rho - \rho r + \rho r\theta]1\]
\[\Rightarrow r - r\theta - \rho - \rho r + \rho r\theta]0\]
\[\Rightarrow r(1 - \theta) - \rho(1 + r - r\theta)]0\]
\[\Rightarrow r(1 - \theta) - \rho[1 + r(1 - \theta)]\]0\]
\[\Rightarrow (1 - \theta)[r - \rho(1 + r)]\]0\]
\[\Rightarrow r - \rho(1 + r)]0\]
\[\Rightarrow \rho\frac{r}{1 + r}\] (8)

**Appendix C: [Derivation of condition 9]**

\[\frac{a + t - [(1 - \rho)(1 + r(1 - \theta)] + \rho\theta]2}{4b}\] \[
\Rightarrow \frac{a + t - [(1 - \rho)(1 + r(1 - \theta)] + \rho\theta]2}{4b}\] \[
\Rightarrow (1 - \alpha)(a - c)\]

\[\Rightarrow 0)(1 - \alpha)(\bar{a} - \bar{c}) - \frac{a + t - [(1 - \rho)(1 + r(1 - \theta)] + \rho\theta]2}{\bar{a} - \bar{c})^2}\]

\[\Rightarrow 0)(1 - \alpha)(\bar{a} - \bar{c}) - \frac{a + t - [(1 - \rho)(1 + r(1 - \theta)] + \rho\theta]2}{\bar{a} - \bar{c})^2}\]

\[\Rightarrow \alpha\frac{1}{2} - \frac{a + t - [(1 - \rho)(1 + r(1 - \theta)] + \rho\theta]2}{2(\bar{a} - \bar{c})^2}\]

(9)

**Appendix D: [Derivation of condition 10]**

\[\frac{1 - \alpha)(\bar{a} - \bar{c})^2}{4b}\] \[
\Rightarrow (1 - \alpha)(\bar{a} - \bar{c})^2(a - c - t)^2\]

\[\Rightarrow (1 - \alpha)(\bar{a} - \bar{c})^2 - (a - c - t)^20\]

\[\Rightarrow (1 - \alpha) - \frac{(a - c - t)^2}{(\bar{a} - \bar{c})^2}0\]

\[\Rightarrow 1 - \frac{(a - c - t)^2}{(\bar{a} - \bar{c})^2}\alpha\]
\[ \Rightarrow \alpha (1 - \frac{(a-c-t)^2}{(a-c)^2}) \]

(10)

REFERENCES