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Performances of precoders and studies of a new detection/precodertechnical

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Abstract

Pilot decontamination is a major challenge of the Massive MIMO system. Pilot contamination severely limits the performance of the system. In front of the many advantages that massive MIMO offer, it is necessary to study techniques that can suppress them. When talking about decontamination, three techniques are the most popular, non-universal reuse pilot, power allocation and precoding/detection. In this work, we chose to study a new precoding/detection technique to participate in this decontamination. Other research has already proposed the full pilot Zero forcing which is an improvement of the ZF, it removes intra and intercell interference by listening to pilot signaling in the entire network. The new technical that we proposed here is based on the fact of being able to improve the useful signal and to eliminate the interferences.

Keywords:

Massive MIMO;
Spectral efficiency;
Precoder;
Detection;
Contamination.

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1. Introduction

Wireless technology has favorably changed the ways of communication. The time when thecommunication is only possible in predefined places is passed. The services ofcommunications are accessible today wherever we are in the world, without supportthrough the deployment of cellular networks, local area networks and satellite networks [1][2][3][4][5] .

About cellular network, it evolves continuously to meet the rapid increaseof demand for wireless data services whose key parameter to consider is throughput(Bits/s). Higher throughput per zone is traditionally achieved by combining three factors:a larger frequency spectrum, higher cell density and spectral efficiency. This study considers the latter, in particular the Massive Multiple-Input conceptMultiple-Output (MIMO) which is identified as a key to increase spectral efficiency in the order of magnitude of the next generation of mobile network.

Massive MIMO system uses spatial multiplexing to improve efficiencyspectral, the ability of the system to separate users from one another defines its performance. Inthe goal of fully exploiting the benefits of Massive MIMO, a proper allocation ofthe spatial resource is then very indispensable. It corresponds to the choice of precoding anddetection techniques and this requires a good knowledge of these signal treatments.

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2. Research Method

We consider a cellular network where payload data istransmitted with universal time and frequency reuse, each cellis assigned an index in the set$\mathcal{L}$, where the cardinality$|\mathcal{L}|$ isthe number of cells. Each BS in each cell is equipped withan array of M antennas and communicates with K singleantennausers (UEs), out of a set of K_{max}UEs. We areinterested in massive MIMO topologies where M and $K_{max}$are large and fixed, while allUEs have unlimited demand for data. The subset of activeUEs changes over time, thus the namek$\in$ {1,…,K} in cell$\ell \in \mathcal{L}$ is given to different UEs at different times. This model is used to study the average performance for arandom set of interfering UEs.

The time/frequencyresources are divided into frames consisting of$T_C$seconds and W_C Hz. This leaves roomforS= $T_C W_C$ transmission symbols per frame.We assumethat the frame dimensions are such that $T_C$is smaller or equalto the coherence time of allUEs while $W_C$is smaller or equalto the coherence bandwidth of all UEs. Hence, all the channelsare staticwithin the frame.

$h_{jlk} \in \mathbb{C}^N$denotes the channelresponse betweenBS j and UE k in cellℓ in a given frame. These channel responses are drawn as realizations from zeromeancircularly symmetric complex Gaussiandistributions:

$h_{jlk} \sim CN(0, d_j(z_{lk}) I_M)$

Where $I_M$is the M×M identity matrix, $d_j(z)$ gives the variance of the channel attenuationfrom BS jto any UE position z, the value of$d_j(z)$varies slowly over time and frequency, thus we assumethat the value is known at BS j for all l and k and that eachUE knows its value to its serving BS. The BSs are not exchanging any information [7][8][9][10][11][12].

2.1 System model

We consider a cellular network where payload data istransmitted with universal time and frequency reuse, each cellis assigned an index in the set$\mathcal{L}$, where the cardinality$|\mathcal{L}|$ isthe number of cells. Each BS in each cell is equipped withan array of M antennas and communicates with K singleantennausers (UEs), out of a set of K_{max}UEs. We areinterested in massive MIMO topologies where M and $K_{max}$are large and fixed, while allUEs have unlimited demand for data. The subset of activeUEs changes over time, thus the namek$\in$ {1, …, K} in cell$\ell \in \mathcal{L}$ is given to different UEs at different times. This model is used to study the average performance for arandom set of interfering UEs[7][8][9][10][11][12].

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2.2 Uplink

The received UL signal$y_j \in \mathbb{C}^M$at BS j ismodeled as:

$$y_j = \sum_{l \in \mathcal{L}} \sum_{k=1}^{K} \sqrt{p_{lk}}\; h_{jlk} x_{lk} + n_j \qquad (1)$$

Where$x_{lk} \in \mathbb{C}$is the symbol transmitted by UE k in cellℓ ; $E\{|x_{lk}|^2\}=1$, $p_{lk} \geq 0$ isthe ULtransmit power ; $p_{lk} = \frac{\rho}{d_l(z_{lk})}$with$\rho > 0$. Thispower-control policy inverts the average channel attenuation$d_l(z_{lk})$. Hence, this policy guarantees a uniform user experience, savesvaluable energy atUEs. Theadditive noise$n_j \in \mathbb{C}^M$ is modeled as$n_j \sim CN(0, \sigma^2 I_M)$; $\sigma^2$is the noise variance[7][8][9][10][11][12].

2.2.1 Pilot signaling

To perform signal processing, the BS needs CSI or Channel State Information. The estimation is done by sending pilot sequence in UL. Each pilot signal can be represented by a deterministic vector$v \in \mathbb{C}^B$. Assume that pilot signals come from a pilot book V defined by:

$$V = \{v_1, \dots ; v_B\} \text{ où } v_{b_1}^H v_{b_2} = \begin{cases} B, b_1 = b_2 \\ 0, b_1 \neq b_2 \end{cases} \qquad (2)$$

Where $(.)^H$ refers to the transposed conjugate. The B pilot signals form an orthogonal basis.

The pilot signal transmitted by UE k in cell l is noted by $v_{i_{lk}}$; i ∈ {1,…,B} is the index in the pilot book. By transmitting these pilot signals over B symbols in the UL system model, the collective received UL signal at BS j is denoted as $y_j \in \mathbb{C}^{M\times B}$ and given by:

$$Y_j = \sum_{l\in\mathcal{L}} \sum_{k=1}^{K} \sqrt{p_{lk}}\ h_{jlk} v_{i_{lk}}^T + N_j \quad (3)$$

Where $N_j \in \mathbb{C}^{M\times B}$contains the additive noise at the receiver during the pilot signaling.

2.2.2 MMSE estimator (minimum mean squared error)

the effective power-controlled UL channel $h_{jlk}^{eff} = \sqrt{p_{lk}} h_{jlk}$for any UE k ∈ {1,…,K} in all cell l ∈$\mathcal{L}$ is :

$$\hat{h}_{jlk}^{eff} = \frac{d_j(z_{lk})}{d_l(z_{lk})} Y_j (\Psi_j^T)^{-1} v_{i_{lk}}^* \quad (4)$$

Where (.)*denotes the complex conjugate, $\Psi_j \in \mathbb{C}^{B\times}$B is the covariance matrix of the received signal :

$$\Psi_j = \sum_{l\in\mathcal{L}} \sum_{m=1}^{k} \frac{d_j(z_{lm})}{d_l(z_{lm})} v_{i_{lm}} v_{i_{lm}}^H + \frac{\sigma^2}{\rho} I_B \quad (5)$$

The estimation error covariance matrix$C_{jlk} \in \mathbb{C}^{M\times M}$ is given by :

$$C_{jlk} = E\{(h_{jlk}^{eff} - \hat{h}_{jlk}^{eff})(h_{jlk}^{eff} - \hat{h}_{jlk}^{eff})^H\} \quad (6)$$

$$C_{jlk} = \rho \frac{d_j(z_{lk})}{d_l(z_{lk})} \left(1 - \frac{\frac{d_j(z_{lk})}{d_l(z_{lk})} B}{\sum_{l\in\mathcal{L}} \sum_{m=1}^{k} \frac{d_j(z_{lm})}{d_l(z_{lm})} v_{i_{lk}} v_{i_{lm}}^H + \frac{\sigma^2}{\rho}} \right) \quad (7)$$

And the mean-squared error is MSE_{jlk} = tr(C_{jlk}).
The covariance matrix C_{jlk} reveals the causes of estimation errors; it depends on the inverse signal-to-noise ratio (SNR), and on which UEs that use the same pilot signal represented by the products $v_{i_{lk}} v_{i_{lm}}^H$ that are non-zero).

. The ratio $\frac{d_j(z_{lm})}{d_l(z_{lm})}$describes the relative strength of theinterference received at BS j from UE m in cell l; it is almostone for cell-edge UEs of neighboring cells, while it is almostzero when cell l is very distant from BS j [25].

2.23 Achievable UL Spectral Efficiencies

The base station can amplify the signal from the kth terminal and eliminate the interference from the other terminals by multiplying the signal received by a vector $g_{jk}^H \in \mathbb{C}^M$, it's the linear combining: $g_{jk}^H y_j$.

$$g_{jk}^H y_j = \sum_{l\in\mathcal{L}} \sum_{k=1}^{K} \sqrt{p_{lk}}\ g_{jk}^H h_{jlk} x_{lk} + g_{jk}^H n_j \quad (8)$$

$$g_{jk}^H y_j = g_{jk}^H h_{jlk} \sqrt{p_{lk}} x_{lk} + \sum_{\substack{t=1\\t\neq k}}^{K} g_{jk}^H h_{jlt} \sqrt{p_{lt}} x_{lt} + \sum_{\substack{i=1\\i\neq l}}^{L} \sum_{t=1}^{K} g_{jk}^H h_{jit} \sqrt{p_{it}} x_{it} + g_{jk}^H n_j \quad (9)$$

Where $g_{jk}^H h_{jlk} \sqrt{p_{lk}} x_{lk}$is the desired signal, $\sum_{\substack{t=1\\t\neq k}}^{K} g_{jk}^H h_{jlt} \sqrt{p_{lt}} x_{lt}$ is the intracell interference, $\sum_{\substack{i=1\\i\neq l}}^{L} \sum_{t=1}^{K} g_{jk}^H h_{jit} \sqrt{p_{it}} x_{it}$ is the intercell interference and $g_{jk}^H n_j$ is the residual noise. The linear combining vector g_{jk}^H appears in all expressions, it can be used to amplify the desired signal, suppress interference and / or suppress noise.
The combining schemes for massive MIMO can have either passive or active interference rejection.. The canonical example of passive rejection is the Maximal Ratio combining (MR), which amplifies the desired signal. On the other hand, the active rejection is obtained by putting the combination at the reception as orthogonal as possible with the interfering channels. This can be achieved through the use of the Zero Forcing combining (ZF), where it is selected to orthogonalize the K intracellular channels.
ZF combining only actively suppresses intra-cell interference, while the inter-cell interference is passively suppressedjust as in MR combining. Further interference rejection canbe achieved by coordinating the combining across cells, suchthat both intra-cell and inter-cell interference are activelysuppressed by the receive combining. [15] proposed a new technique, fullpilot zero-forcing (P-ZF) defined as:

$$g_{jk}^{P-ZF} = \hat{H}_{v,j} (\hat{H}_{v,j}^H \hat{H}_{v,j})^{-1} e_{i_{jk}} \quad (10)$$

P-ZF exploits that all the B estimated channel directions $\hat{H}_{v,j}$ are known at BS j and orthogonalizes all these directions to also mitigate parts of the inter-cell interference.
In the UL, an ergodic achievable SE of an arbitraryUE k in cell j is:
$\zeta^{(ul)}\left(1-\frac{B}{s}\right)E_{\{z\}}\left\{log_2\left(1+SINR_{jk}^{(ul)}\right)\right\}$[bit/s/Hz]
Where :

$$SINR_{jk}^{(ul)}=\frac{p_{jk}\left|E_{\{h\}}\{g_{jk}^H h_{jjk}\}\right|^2}{\sum_{l\in\mathcal{L}}\sum_{m=1}^{K}p_{lm}E_{\{h\}}\{|g_{jk}^H h_{jlm}|^2\}-p_{jk}\left|E_{\{h\}}\{g_{jk}^H h_{jjk}\}\right|^2+\sigma^2 E_{\{h\}}\{||g_{jk}||^2\}} \quad (11)$$

Let $L_j(\beta)\subset L$ be the subset of cells that uses the same pilots as cell j. In the UL, an achievable SE in cell j is:

$$SE_j^{(ul)}=K\zeta^{(ul)}(1-\frac{B}{S})\log_2(1+\frac{1}{I_j^{scheme}})\text{ [bit/s/Hz/cell]} \quad (12)$$

Where the interference term:

$$I_j^{scheme}=\sum_{l\in\mathcal{L}_j(\beta)\setminus\{j\}}\left(\mu_{jl}^{(2)}+\frac{\mu_{jl}^{(2)}-\left(\mu_{jl}^{(1)}\right)^2}{G^{scheme}}\right)+\frac{(\sum_{l\in\mathcal{L}}\mu_{jl}^{(1)}Z_{jl}^{scheme}+\frac{\sigma^2}{\rho})(\sum_{l\in\mathcal{L}_j(\beta)}\mu_{jl}^{(1)}+\frac{\sigma^2}{B\rho})}{G^{scheme}} \quad (13)$$

depends on the receive combining scheme throughG^{scheme} and Z_{jl}^{scheme} ;

$$\mu_{jl}^{(w)}=E_{z_{lm}}\left\{\left(\frac{d_j(z_{lm})}{d_l(z_{lm})}\right)^w\right\}\text{ pour w=1,2.} \quad (14)$$

Achievable SE in UL depends on the combining scheme.

2.3 Calculation of the interference term I_j^{scheme}

We have :

$$I_j^{scheme}=E\left\{\frac{1}{SINR_{jk}^{(ul)}}\right\} \quad (15)$$

And $SINR_{jk}^{(ul)}=\frac{p_{jk}\left|E_{\{h\}}\{g_{jk}^H h_{jjk}\}\right|^2}{\sum_{l\in\mathcal{L}}\sum_{m=1}^{K}p_{lm}E_{\{h\}}\{|g_{jk}^H h_{jlm}|^2\}-p_{jk}\left|E_{\{h\}}\{g_{jk}^H h_{jjk}\}\right|^2+\sigma^2 E_{\{h\}}\{\left||g_{jk}|\right|^2\}}$

For MR :

$$SINR_{jk}^{MR}=\frac{v_{i_{jk}}^H\Psi_j^{-1}v_{i_{jk}}}{\sum_{l\in L}\sum_{m=1}^{K}\left(\frac{d_j(z_{lm})}{d_l(z_{lm})}\frac{1}{M}+\left(\frac{d_j(z_{lm})}{d_l(z_{lm})}\right)^2 v_{i_{jk}}^H\Psi_j^{-1}v_{i_{lm}}\right)-v_{i_{jk}}^H\Psi_j^{-1}v_{i_{jk}}+\frac{\sigma^2}{M\rho}} \quad (16)$$

$$\frac{1}{SINR_{jk}^{MR}}=\frac{\sum_{l\in L}\sum_{m=1}^{K}\left(\frac{d_j(z_{lm})}{d_l(z_{lm})}\frac{1}{M}+\left(\frac{d_j(z_{lm})}{d_l(z_{lm})}\right)^2 v_{i_{jk}}^H\Psi_j^{-1}v_{i_{lm}}\right)-v_{i_{jk}}^H\Psi_j^{-1}v_{i_{jk}}+\frac{\sigma^2}{M\rho}}{v_{i_{jk}}^H\Psi_j^{-1}v_{i_{jk}}}$$

$I_j^{MR}=E_{\{z\}}\left\{\frac{1}{SINR_{jk}^{MR}}\right\}$:

$$E_{\{z\}}\left\{\frac{1}{v_{i_{jk}}^H\Psi_j^{-1}v_{i_{jk}}}\right\}=E_{\{z\}}\left\{\frac{\sum_{l\in L}\sum_{m=1}^{K}v_{i_{jk}}^H\Psi_j^{-1}v_{i_{lm}}+\frac{\sigma^2}{\rho}}{B}\right\}=\frac{\sum_{l\in L_j}\mu_{jl}^{(1)}B+\frac{\sigma^2}{\rho}}{B}$$

$$E_{\{z\}}\left\{\sum_{l\in L}\sum_{m=1}^{K}\left(\frac{d_j(z_{lm})}{d_l(z_{lm})}\right)^2\frac{v_{i_{jk}}^H\Psi_j^{-1}v_{i_{lm}}}{v_{i_{jk}}^H\Psi_j^{-1}v_{i_{jk}}}\right\}=\sum_{l\in L_j}\mu_{jl}^{(2)}$$

$$E_{\{z\}}\left\{\sum_{l\in L}\sum_{m=1}^{K}\frac{\frac{d_j(z_{lm})}{d_l(z_{lm})}}{v_{i_{jk}}^H\Psi_j^{-1}v_{i_{jk}}}\right\}=\sum_{l\in L}K\mu_{jl}^{(1)}\frac{\sum_{l\in L_j}\mu_{jl}^{(1)}B+\frac{\sigma^2}{\rho}}{B}+\sum_{l\in L_j}\mu_{jl}^{(2)}-\left(\mu_{jl}^{(1)}\right)^2$$

$$I_j^{MR}=\sum_{l\in\mathcal{L}_j(\beta)\setminus\{j\}}\left(\mu_{jl}^{(2)}+\frac{\mu_{jl}^{(2)}-\left(\mu_{jl}^{(1)}\right)^2}{M}\right)+\frac{(\sum_{l\in\mathcal{L}}\mu_{jl}^{(1)}K+\frac{\sigma^2}{\rho})(\sum_{l\in\mathcal{L}_j(\beta)}\mu_{jl}^{(1)}+\frac{\sigma^2}{B\rho})}{M}$$

With the same steps, we get:

$$I_j^{ZF} = \sum_{l\in\mathcal{L}_j(\beta)\setminus\{j\}} \left(\mu_{j\,l}^{(2)} + \frac{\mu_{j\,l}^{(2)} - \left(\mu_{jl}^{(1)}\right)^2}{M-K}\right) + \frac{\left(\sum_{l\in\mathcal{L}} \mu_{jl}^{(1)} K\left(1 - \frac{\mu_{j\,l}^{(1)}}{\sum_{l\in\mathcal{L}_j(\beta)} \mu_{jl}^{(1)} + \frac{\sigma^2}{B\rho}}\right) + \frac{\sigma^2}{\rho}\right)\left(\sum_{l\in\mathcal{L}_j(\beta)} \mu_{jl}^{(1)} + \frac{\sigma^2}{B\rho}\right)}{M-K}$$

$$I_j^{P-ZF} = \sum_{l\in\mathcal{L}_j(\beta)\setminus\{j\}} \left(\mu_{j\,l}^{(2)} + \frac{\mu_{j\,l}^{(2)} - \left(\mu_{jl}^{(1)}\right)^2}{M-B}\right) + \frac{\left(\sum_{l\in\mathcal{L}} \mu_{jl}^{(1)} K\left(1 - \frac{\mu_{j\,l}^{(1)}}{\sum_{l\in\mathcal{L}(\beta)} \mu_{jl}^{(1)} + \frac{\sigma^2}{B\rho}}\right) + \frac{\sigma^2}{\rho}\right)\left(\sum_{l\in\mathcal{L}_j(\beta)} \mu_{jl}^{(1)} + \frac{\sigma^2}{B\rho}\right)}{M-B}$$

$$I_j^{scheme} = \sum_{l\in\mathcal{L}_j(\beta)\setminus\{j\}} \left(\mu_{j\,l}^{(2)} + \frac{\mu_{j\,l}^{(2)} - \left(\mu_{jl}^{(1)}\right)^2}{G^{scheme}}\right) + \frac{\left(\sum_{l\in\mathcal{L}} \mu_{jl}^{(1)} Z_{jl}^{scheme} + \frac{\sigma^2}{\rho}\right)\left(\sum_{l\in\mathcal{L}_j(\beta)} \mu_{jl}^{(1)} + \frac{\sigma^2}{B\rho}\right)}{G^{scheme}}$$

So,
MR combining is obtained by:

$G^{MR}=M$ ***(17)***

$$Z_{jl}^{MR} = K \qquad (18)$$

while ZF combining is obtained by:

$G^{ZF}= M-K$ ***(19)***

$$Z_{jl}^{ZF} = \begin{cases} K\left(1 - \frac{\mu_{j\,l}^{(1)}}{\sum_{l\in\mathcal{L}_j(\beta)} \mu_{jl}^{(1)} + \frac{\sigma^2}{B\rho}}\right) si\, l \in \mathcal{L}_j(\beta) \\ K si\, l \notin \mathcal{L}_j(\beta) \end{cases} \qquad (20)$$

And P-ZF is obtained by:

$G^{P-ZF}= M-B$ ***(21)***

$$Z_{jl}^{P-ZF} = K\left(1 - \frac{\mu_{j\,l}^{(1)}}{\sum_{l\in\mathcal{L}_l(\beta)} \mu_{j\,l}^{(1)} + \frac{\sigma^2}{B\rho}}\right) \qquad (22)$$

2.4Downlink

Building on the UL/DL channel reciprocity in calibrated TDD systems, the received DL signal $z_{jk}\in\mathbb{C}$ at l'UE k in cell j is modeled as:

$$z_{jk}=\sum_{l\in\mathcal{L}} \sum_{m=1}^{k} h_{ljk}^T\, w_{lm} s_{lm} + n_{jk} \qquad (23)$$

Where $(.)^T$ denotes transpose, s_{lm} is the symbol intended for UE m in cell l, $W_{lm}\in\mathbb{C}M$ is the vector of precoding and $||W_{lm}||^2$ is the allocated DL transmit power.

Power control can be considered in the DL since the BS has access to the estimated CSI[7][8][9][10][11][12]. Channel estimation is also used for DL linear precoding where the M channel inputs are utilized to make each data signal add up coherently at its desired UE and to suppress the interference caused to other UEs.

$W_{jk}\in\mathbb{C}^M$ is the precoding vector associated with UE k in cell j. We express these precodingvectors as :

$$W_{jk} = \sqrt{\frac{q_{jk}}{E_{\{h\}}\left\{||\check{g}_{jk}||^2\right\}}}\, \check{g}_{jk}^*$$

Where $q_{jk} \geq 0$ is the emission average power, $\check{g}_{jk} \in\mathbb{C}^M$ defines the spatial directivity of the transmission and is based on the acquired CSI.

We will see later how to choose the emission power in DL to have the same efficiency as in UL. n_{jk} is the additive noise to the UE k in cell j and $n_{jk} \sim CN(0, \sigma^2)$ with the same variance as in uplink. There is no pilot transmission in DL because the response of the channel is the same as that in UL, it is channel reciprocity.

2.4.1 Achievable DL spectral efficiencies

In the DL, an ergodic achievable SE of an arbitrary UE k in cell j is:

$\zeta^{(dl)}\left(1-\frac{B}{s}\right)E_{\{z\}}\left\{log_2\left(1+SINR_{jk}^{(dl)}\right)\right\}$ [bit/s/Hz]

Where:

$$SINR_{jk}^{(dl)}=\frac{q_{jk}\frac{\left|E_{\{h\}}\{\breve{g}_{jk}^{H}h_{jjk}\}\right|^2}{E_{\{h\}}\{\|\breve{g}_{jk}\|^2\}}}{\sum_{l\in\mathcal{L}}\sum_{m=1}^{K}q_{lm}\frac{E_{\{h\}}\left\{\left|\breve{g}_{lm}^{H}h_{ljk}\right|^2\right\}}{E_{\{h\}}\{\|\breve{g}_{lm}\|^2\}}-q_{jk}\frac{\left|E_{\{h\}}\{\breve{g}_{jk}^{H}h_{jjk}\}\right|^2}{E_{\{h\}}\{\|\breve{g}_{jk}\|^2\}}+\sigma^2} \quad \textbf{(24)}$$

This equation shows that the UE knows only the expectations $E_{\{h\}}$ but does not know the realization of channel h.

2.4.2 Duality UL-DL

There is a strong connection between the transmission precoding and the receive combining. Let $\{g_{jk}^{scheme}\}$ be the set of combining vector in UL. So, there is a DL power control policy $\{q_{jk}\}$ with $\sum_{j\in\mathcal{L}}\sum_{k=1}^{K}q_{jk}=\sum_{j\in\mathcal{L}}\sum_{k=1}^{K}p_{jk}$ for $SINR_{jk}^{(dl)}=SINR_{jk}^{(ul)}$ using $\breve{g}_{jk}=g_{jk}^{scheme}$ for all j and k. Therefore, the same spectral efficiency can be obtained in UL and DL:

$$SE_j^{(dl)}=K\zeta^{(dl)}\left(1-\frac{B}{S}\right)\log_2\left(1+\frac{1}{I_j^{scheme}}\right)\text{[bit/s/Hz/cell]} \quad \textbf{(25)}$$

Where the interference term I_j^{scheme} in DL is the same as in UL.

This equation shows that the SINR obtained in the uplink is also obtained in the downlink by correctly choosing the power control coefficient $\{q_{jk}\}$.

Hence the conventionally known DL-UL duality for single cell systems with a perfect CSI is also applicable in the Massive MIMO multi-cellular system with an estimated CSI. The same treatment is used in UL and DL.

2.5 Studies of new detection/precoder in terms of spectral efficiency (SE)

MR maximizes the desired signal but does not suppress intracellular interference, ZF suppresses intracellular interference and P-ZF suppresses intra and intercell interference by listening to all pilot sequences throughout the network [4] [5] [13] [14]. Now, we will study a new signal processing technique that maximizes the desired signal while trying to suppress interference. We will call it 'VAO'.

The spectral efficiency of a cell which K UEs are active is:

$$SE_j=K\left(1-\frac{B}{S}\right)\log_2\left(1+\frac{1}{I_j^{scheme}}\right)\text{[bit/s/Hz/cell]} \quad \textbf{(26)}$$

Let's study the achievable SE with VAO.

2.5.1Detection/precoding technique VAO

It uses the channel state estimate to detect the received signal and to direct the signal to the intended UE. After estimating the channel state, the signal is multiplied with the processing vector.

It is of the form:

$$g^{VAO}=S^{-1}\hat{H} \quad \textbf{(27)}$$

With $S=\hat{H}\hat{H}^H+\sigma^2$

The result of signal processing is:

$$G^Hy=\sqrt{p}G^H\hat{H}x-\sqrt{p}G^H\tilde{H}x+G^Hn \quad \textbf{(28)}$$

$$G^Hy=\sqrt{p}(S^{-1}\hat{H})^H\hat{H}x-\sqrt{p}(S^{-1}\hat{H})^H\tilde{H}x+G^Hn \quad \textbf{(29)}$$

The desired signal is :

$$\sqrt{p}(S^{-1}\hat{H})^H\hat{H}x=\sqrt{p}((\hat{H}\hat{H}^H+\sigma^2)^{-1}\hat{H})^H\hat{H}x \quad \textbf{(30)}$$

The interference term is a random variable of zero mean and variance:

$$var\{(S^{-1}\hat{H})^H(n-\sqrt{p}\tilde{H}x)\}=var\left\{((\hat{H}\hat{H}^H+\sigma^2)^{-1}\hat{H})^H(n-\sqrt{p}\tilde{H}x)\right\} \quad \textbf{(31)}$$

With

C_{jlk}= E$\{(h_{jlk}^{eff} - \hat{h}_{jlk}^{eff})(h_{jlk}^{eff} - \hat{h}_{jlk}^{eff})^H\}$= E$\{\tilde{H}\tilde{H}^H\}$

C_{jlk}= $\rho\frac{d_j(z_{lk})}{d_l(z_{lk})}\left(1-\frac{\frac{d_j(z_{lk})}{d_l(z_{lk})}B}{\sum_{l\in\mathcal{L}}\sum_{m=1}^{k}\frac{d_j(z_{lm})}{d_l(z_{lm})}v_{i_{lk}}v_{i_{lm}}^H+\frac{\sigma^2}{\rho}}\right)$

2.5.2 Calculation of SINR

Channel estimation and the use of signal processing as VAO in Massive MIMO allow to obtain a better SINR.

$$SINR_{jk}^{(ul)}=\frac{p_{jk}\left|E_{\{h\}}\{g_{jk}^H h_{jjk}\}\right|^2}{\sum_{l\in\mathcal{L}}\sum_{m=1}^{K}p_{lm}E_{\{h\}}\{|g_{jk}^H h_{jlm}|^2\}-p_{jk}\left|E_{\{h\}}\{g_{jk}^H h_{jjk}\}\right|^2+\sigma^2 E_{\{h\}}\{||g_{jk}||^2\}}$$

Where:

$$E_{\{h\}}\{||g_{jk}||^2\}=\frac{v_{i_{jk}}^H\Psi_j^{-1}v_{i_{jk}}}{M\rho}$$

$$p_{jk}\left|E_{\{h\}}\{g_{jk}^H h_{jjk}\}\right|^2=1$$

$$\sum_{l\in\mathcal{L}}\sum_{m=1}^{K}p_{lm}E_{\{h\}}\{|g_{jk}^H h_{jlm}|^2\}=\frac{\left(\frac{d_j(z_{lm})}{d_l(z_{lm})}+\frac{d_j(z_{lm})}{d_l(z_{lm})}v_{i_{lm}}^H\Psi_j^{-1}v_{i_{lm}}\right)+\left(\frac{d_j(z_{lm})}{d_l(z_{lm})}\right)^2 v_{i_{jk}}^H\Psi_j^{-1}v_{i_{lm}}}{M\rho v_{i_{jk}}^H\Psi_j^{-1}v_{i_{jk}}}$$

So :

$$SINR_{jk}^{VAO}=\frac{v_{i_{jk}}^H\Psi_j^{-1}v_{i_{jk}}}{\sum_{l\in L}\sum_{m=1}^{K}\left(\left(\frac{d_j(z_{lm})}{d_l(z_{lm})}+\frac{d_j(z_{lm})}{d_l(z_{lm})}v_{i_{lm}}^H\Psi_j^{-1}v_{i_{lm}}\right)\frac{1}{M}+\left(\frac{d_j(z_{lm})}{d_l(z_{lm})}\right)^2 v_{i_{jk}}^H\Psi_j^{-1}v_{i_{lm}}\right)-v_{i_{jk}}^H\Psi_j^{-1}v_{i_{jk}}+\frac{\sigma^2}{M\rho}}$$

$$SINR_{jk}^{VAO}=\frac{1}{\frac{\sum_{l\in L}\sum_{m=1}^{K}\left(\left(\frac{d_j(z_{lm})}{d_l(z_{lm})}+\frac{d_j(z_{lm})}{d_l(z_{lm})}v_{i_{lm}}^H\Psi_j^{-1}v_{i_{lm}}\right)\frac{1}{M}+\left(\frac{d_j(z_{lm})}{d_l(z_{lm})}\right)^2 v_{i_{jk}}^H\Psi_j^{-1}v_{i_{lm}}\right)-v_{i_{jk}}^H\Psi_j^{-1}v_{i_{jk}}+\frac{\sigma^2}{M\rho}}{v_{i_{jk}}^H\Psi_j^{-1}v_{i_{jk}}}}$$

$$SINR_{jk}^{VAO}=\frac{1}{\left(\sum_{l\in L}\sum_{m=1}^{K}\frac{d_j(z_{lm})}{d_l(z_{lm})}+\frac{d_j(z_{lm})}{d_l(z_{lm})}v_{i_{lm}}^H\Psi_j^{-1}v_{i_{lm}}\right)\frac{1}{Mv_{i_{jk}}^H\Psi_j^{-1}v_{i_{jk}}}-1+\frac{\sigma^2}{M\rho v_{i_{jk}}^H\Psi_j^{-1}v_{i_{jk}}}++\sum_{l\in L}\sum_{m=1}^{K}\left(\frac{d_j(z_{lm})}{d_l(z_{lm})}\right)^2\frac{v_{i_{lm}}}{v_{i_{jk}}}} \quad (32)$$

VAO takes into account all interfering signals from other cells by S^{-1} which acts as a filter. It maximizes the SINR by seeking a balance between the amplification of the desired signal and the suppression of interference in the spatial domain. Its disadvantage is the complexity of the inverse matrix calculation.

2.5.3Calculation of interference term for VAO

The interference term informs about pilot contamination and interference between users.

$$I_j^{scheme}=E\left\{\frac{1}{SINR_{jk}^{(ul)}}\right\}$$

$$\frac{1}{SINR_{jk}^{(ul)}}=\left(\frac{\sum_{l\in L}\sum_{m=1}^{K}\frac{d_j(z_{lm})}{d_l(z_{lm})}+\frac{d_j(z_{lm})}{d_l(z_{lm})}v_{i_{lm}}^H\Psi_j^{-1}v_{i_{lm}}}{Mv_{i_{jk}}^H\Psi_j^{-1}v_{i_{jk}}}\right)-1+\frac{\sigma^2}{M\rho v_{i_{jk}}^H\Psi_j^{-1}v_{i_{jk}}}$$
$$++\sum_{l\in L}\sum_{m=1}^{K}\left(\frac{d_j(z_{lm})}{d_l(z_{lm})}\right)^2\frac{v_{i_{lm}}}{v_{i_{jk}}}$$

$$I_{jk}^{VAO}=\sum_{l\in\mathcal{L}_j(\beta)\backslash\{j\}}\left(\mu_{j_l}^{(2)}+\frac{\mu_{j_l}^{(2)}-\left(\mu_{jl}^{(1)}\right)^2}{M}\right)+\frac{\left(\sum_{l\in\mathcal{L}}\mu_{jl}^{(1)}\left(K+K\left(1-\frac{\mu_{j_l}^{(1)}}{\sum_{l\in\mathcal{L}_j(\beta)}\mu_{jl}^{(1)}+\frac{\sigma^2}{B\rho}}\right)\right)+\frac{\sigma^2}{\rho}\right)\left(\sum_{l\in\mathcal{L}_j(\beta)}\mu_{jl}^{(1)}+\frac{\sigma^2}{B\rho}\right)}{M}$$

$$I_{jk}^{VAO} = \sum_{l\in\mathcal{L}_j(\beta)\setminus\{j\}} \left(\mu_{j_l}^{(2)} + \frac{\mu_{j_l}^{(2)} - \left(\mu_{jl}^{(1)}\right)^2}{M}\right) + \frac{(\sum_{l\in\mathcal{L}} \mu_{jl}^{(1)} K\left(2 - \frac{\mu_{j_l}^{(1)}}{\sum_{l\in\mathcal{L}_j(\beta)} \mu_{jl}^{(1)} + \frac{\sigma^2}{B\rho}}\right) + \frac{\sigma^2}{\rho})(\sum_{l\in\mathcal{L}_j(\beta)} \mu_{jl}^{(1)} + \frac{\sigma^2}{B\rho})}{M} \quad (33)$$

VAO is obtained with:

$$G^{VAO} = M \quad (34)$$

$$Z_{jl}^{VAO} = \begin{cases} K\left(2 - \frac{\mu_{j_l}^{(1)}}{\sum_{l\in\mathcal{L}_j(\beta)} \mu_{jl}^{(1)} + \frac{\sigma^2}{B\rho}}\right) if\ l \in \mathcal{L}_j(\beta) \\ Ksil \notin \mathcal{L}_j(\beta) \end{cases} \quad (35)$$

Maximizing the SINR consists in suppressing the interferences between the signals and amplifying the level of the useful signal. And increasing the SINR allows an improvement of the spectral efficiency since the ES is a logarithmic function of the SINR.

2.6 Asymptotic analysis

Let $L_j(\beta)\subset L$ be the subset of cells that uses the same pilot as cell j[4] [5] [13] [14].

If $M \to \infty$ with K, $B \leq S < \infty$, the effective SINRs with MR, ZF , P-ZF and VAO converge to the same limit : $\frac{1}{I_j^{MR}}$, $\frac{1}{I_j^{ZF}}$, $\frac{1}{I_j^{P-ZF}}$, $\frac{1}{I_j^{VAO}} \to \frac{1}{\sum_{l\in\mathcal{L}_j(\beta)\setminus\{j\}} \mu_{jl}^{(2)}}$

Only cells using the same pilot as cell j affect the asymptotic limit. To maximize the asymptotic SINR, we must place the cells with a large $\mu_{jl}^{(2)}$ in a different subset so that they use different pilot.

To find the optimal K, we can use the asymptotic limit as follows:

$$SE_j^{\infty} = K\left(1 - \frac{K\beta}{S}\right)\log_2\left(1 + \frac{1}{\sum_{l\in\mathcal{L}_j(\beta)\setminus\{j\}} \mu_{jl}^{(2)}}\right);\ M \to \infty \quad (36)$$

This SE is maximized jointly for all cells when the number of scheduled UEs is either $K^* = \left\lfloor \frac{S}{2\beta} \right\rfloor$ or $K^* = \left\lceil \frac{S}{2\beta} \right\rceil$ that is to say one of the closest integers to $\frac{S}{2\beta}$. With $K^*$is the optimal number of scheduled UEs.

Hence the optimal asymptotic SE is:

$$SE_j^{\infty} = \frac{S}{4\beta}\log_2\left(1 + \frac{1}{\sum_{l\in\mathcal{L}_j(\beta)\setminus\{j\}} \mu_{jl}^{(2)}}\right) \quad (37)$$

3. Results and Analysis of different precoders/detections

In the following simulations, we will consider decoding/combining techniques MR, ZF, P-ZF and VAO; all the results are obtained by using the expressions mentioned above. Curves are obtained using Matlab software.

For each number of antennas M, the spectral efficiency with respect to the number of terminals K and the pilot reuse factor β was calculated by searching the range of any reasonable integer value which determines the length of the pilot B (B = βK). Let S = 400 be the length of the coherence block, SNR = 5dB. The impacts of the parameter change will be studied afterwards.

3.1 Achievanle SE with different precoders/detections

The result is for mean interference severity, this is a practical case since the average comes from the mobility of the UEs, the precoding-combining and the random pilot sequence exchange between the UEs in each cell (all UEs can not be at the same time in the worst locations compared to other cells). The result is shown in Figure 1.

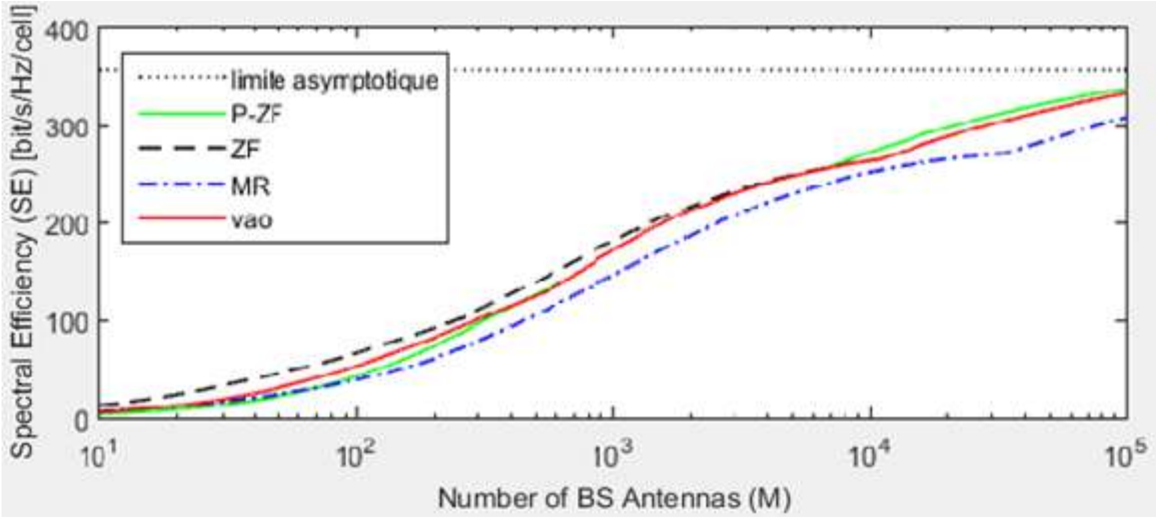

Fig 1: SE depending on the number of BS antennas

The asymptotic limit shows the achievable SE when the SINR is maximal, with M tends to infinity. It is used as the upper limit for ES measurement.

The spectral efficiencies obtained show great differences for the different precoders. MR achieves the lowest ES, this is due to the fact that it does not suppress interference. ZF is the most efficient when M is less than 1000 with less interference and for M between 500 and 8000, VAO and P-ZF have the same performance. Beyond that, P-ZF is the most powerful; it exceeds VAO when M is very large. The performance of P-ZF requires a very high number of antennas from the base station. At least M = 10^5 is necessary to reach the asymptotic limit.

3.2 Optimal number of scheduled UEs depending of M

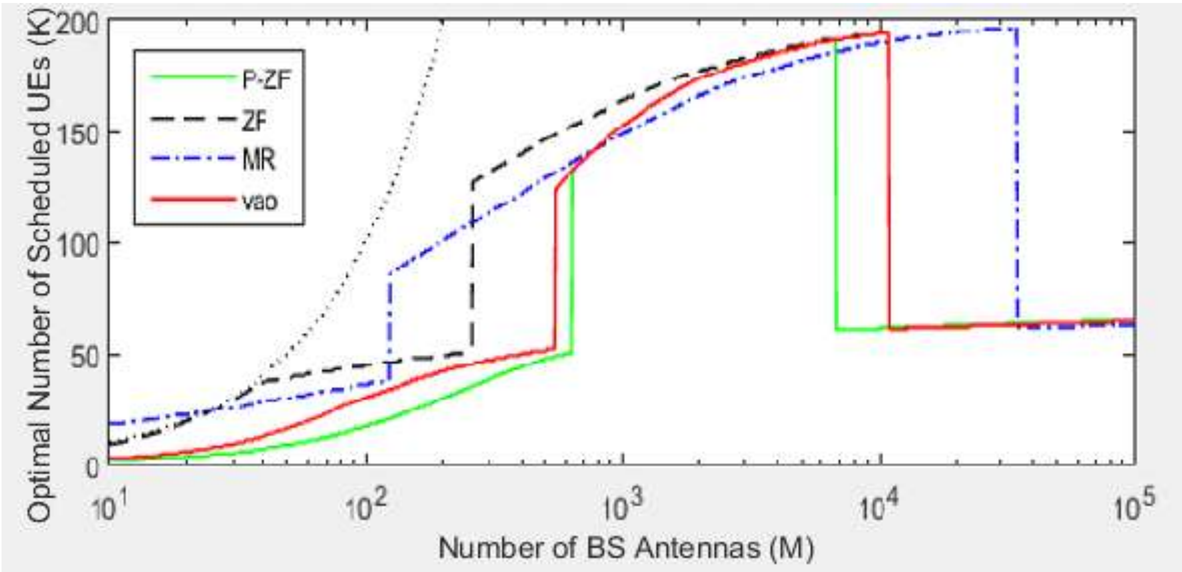

Fig 2: Optimal number of UE depending M

The general behavior is that larger M implies an optimal number of UEs, K* higher. MR, ZF, P-ZF and VAO plan the same optimal number of UEs, around 180 when M is very large. MR can schedule the largest number of UEs with the smallest number of antennas at the BS (M = 150, MR plans 80 UEs).

3.3 Impacts of changing system parameters

Let's look at how the system settings affect the results of the simulation. Let's focus on the case where 10≤M ≤1000 antennas and also when other parameters of the system than M vary. We consider the average Massive MIMO configuration (M = 100) and the large Massive MIMO configuration (M = 500.

- Per user SE

Consider the SE per UE for the operating points that maximize the SE per cell, this is the ES/K* ratio.

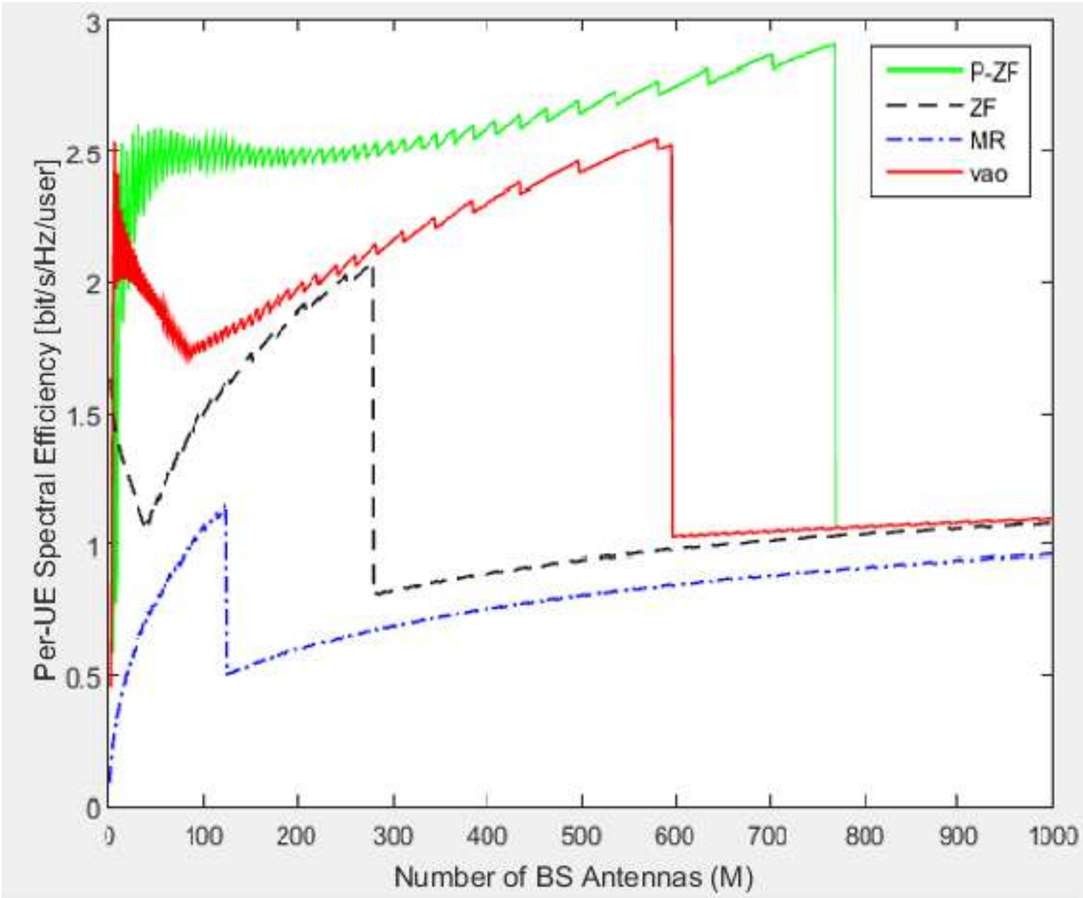

Fig 3:Per user SE

It note that MR gives the lowest SE per scheduled UE while P-ZF gives the highest SE when M is large. The values are around1bit/s/Hz/UE for MR, in the range of 1-2,5 bit/s/Hz/UE for ZF and VAO and in the range of 1-3bit/s/Hz/UE for P- ZF.

- BS antennas per UE

Take the M/K* ratio, this ratio can be interpreted as the number of BS antennas per UE.

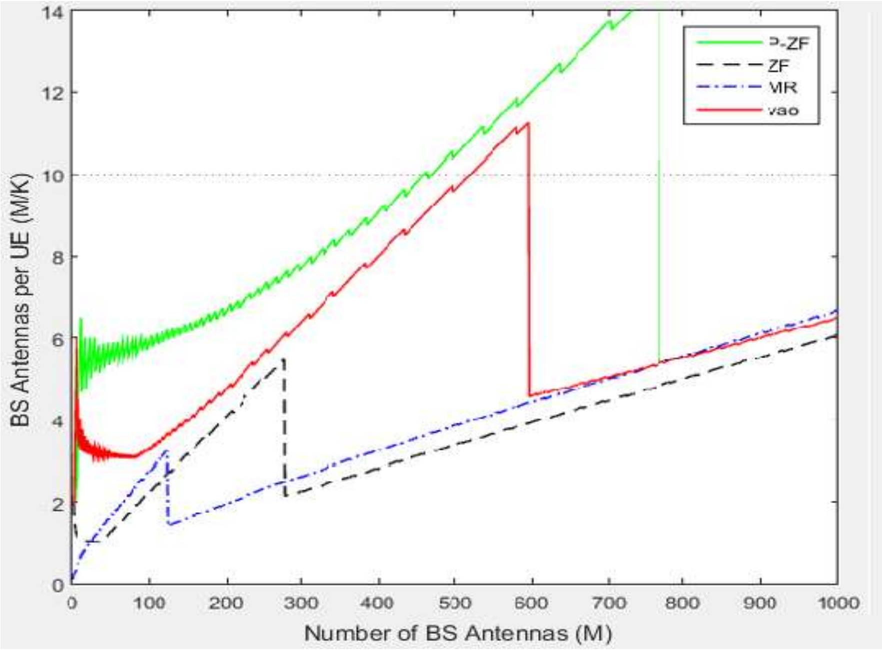

Fig 4:BS antennas per UE

There is a common rule of thumb that says Massive MIMO systems should have an order of magnitude of BS antennas larger than the number of UEs. The operating points that satisfy this directive are above the horizontal dotted line (10 times more BS antennas than UEs). This simulation indicates that an optimized system might not follow this guideline: in fact, there are some occasions where MR even prefers to have M/K* < 1. In general, it seems that 2 to 8 times more of BS antennas than UEs is the range to aim for practical deployments.

- Number of scheduled UE

Does a higher SE imply that the number of scheduled UEs is also higher? Figure 5 allows us to answer this question.

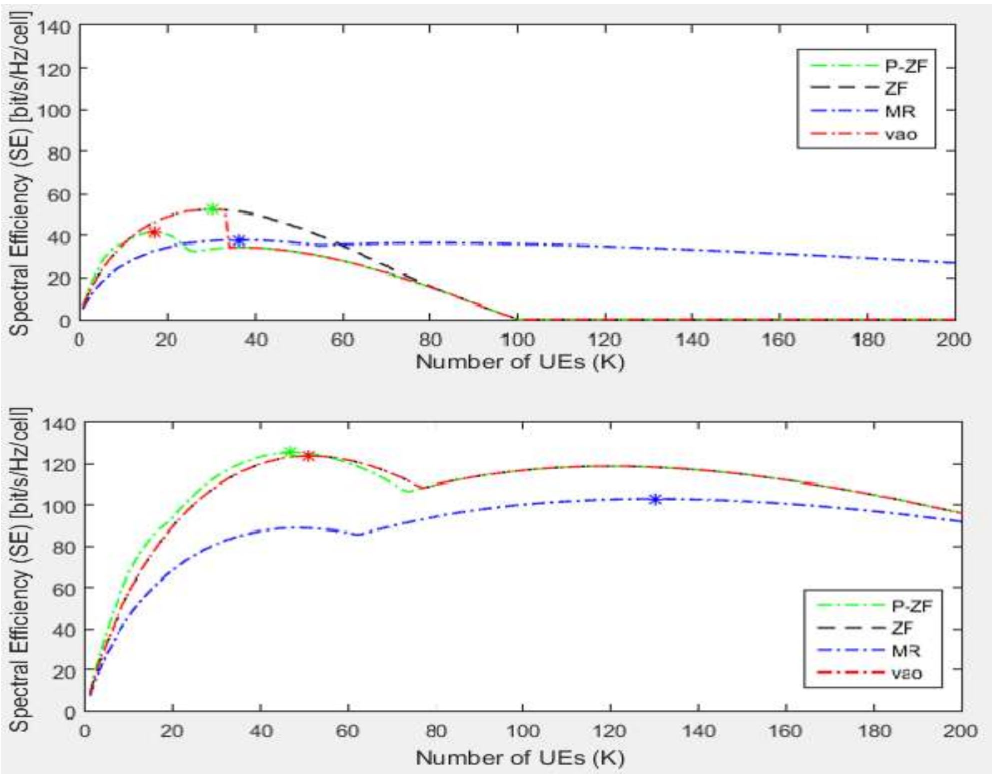

Fig 5:SE depending of K

Since the cells may not be fully charged at each moment, noted that for each scheme, the peaks are at K different. For a given K, the differences between the MR, ZF, P-ZF and VAO schemes can be either larger or smaller at the peaks. Although ZF, P-ZF, and VAO often provide a better SE than MR, it is interesting to note that MR is competitive when K is large (the other coding strongly depend on the estimation of the channel state for the suppression of interference). These results are confirmed for the average configuration, M = 100 which is shown by the curves in the first marker in Figure5 and the large configuration of Massive MIMO, M = 500 which is shown by the second marker.

- Average SNR ρ/σ^2

Does the SNR have an impact on SE? To answer this question, let's see Figures 6 and 7.

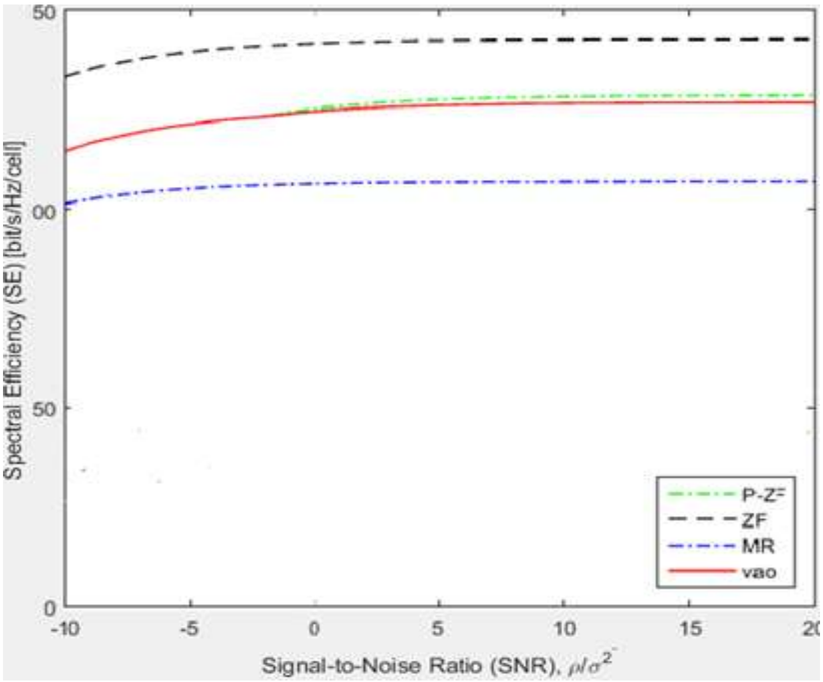

Fig 6: Achievable SE varying SNR for M=500

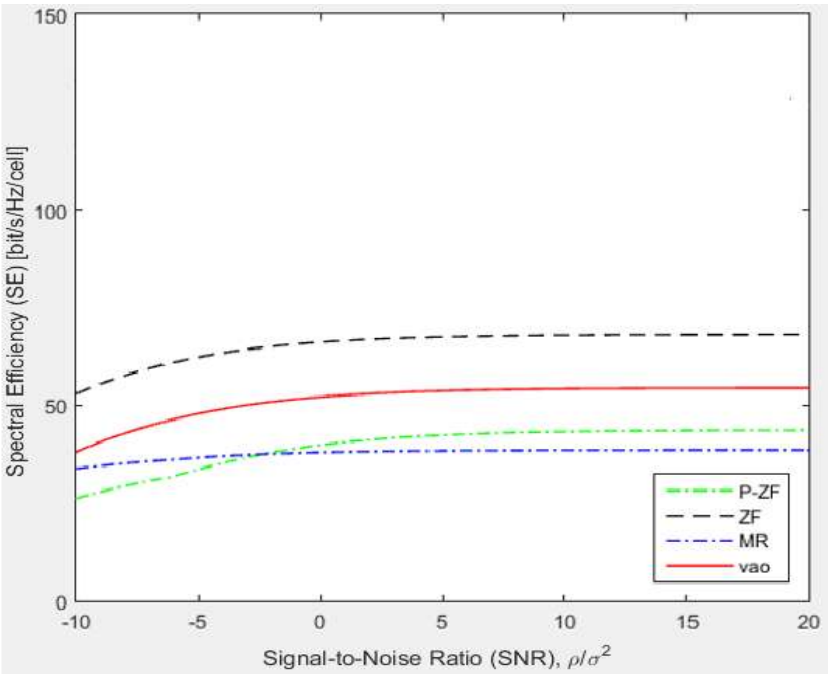

Fig 7:Acheavable SE varying SNR for M=100

Effect of the average SNR ρ/σ^2: the SE is already saturated at a SNR of 5 dB (from 5 dB, the SE does not vary). Massive MIMO can also operate at lower SNRs but with lower performance. ZF, P-ZF and VAO are particularly sensitive to the value of the SNR since the suppression of interference requires a higher quality of channel estimation than the single MR receiver. In addition, MR improves the desired signal level.

- Coherence block length

What effect does the length of the coherence block have on the spectral efficiency of a cell? Let's look at Figures 8 and 9.

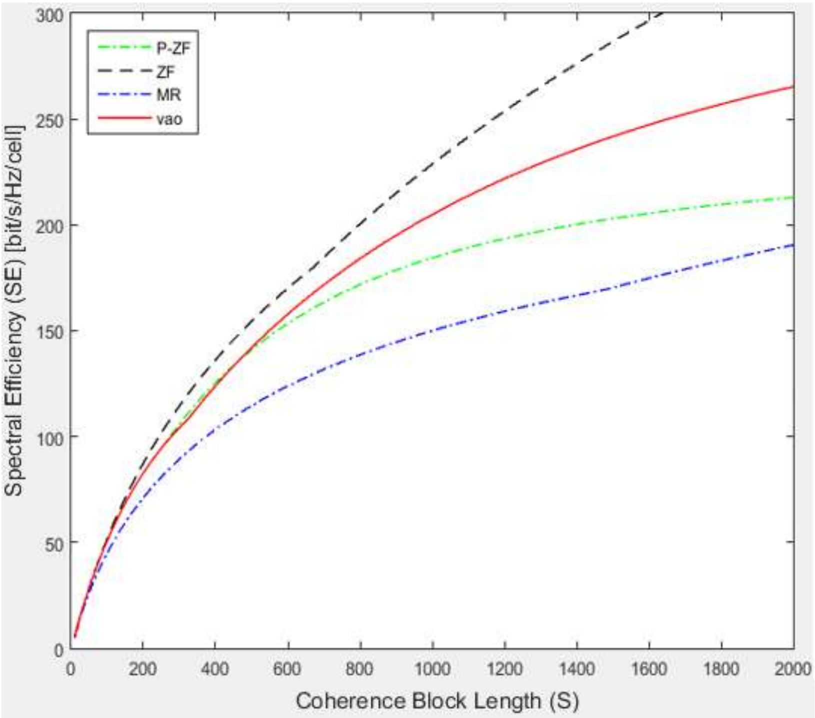

Fig 8:SE depending of S for M=500

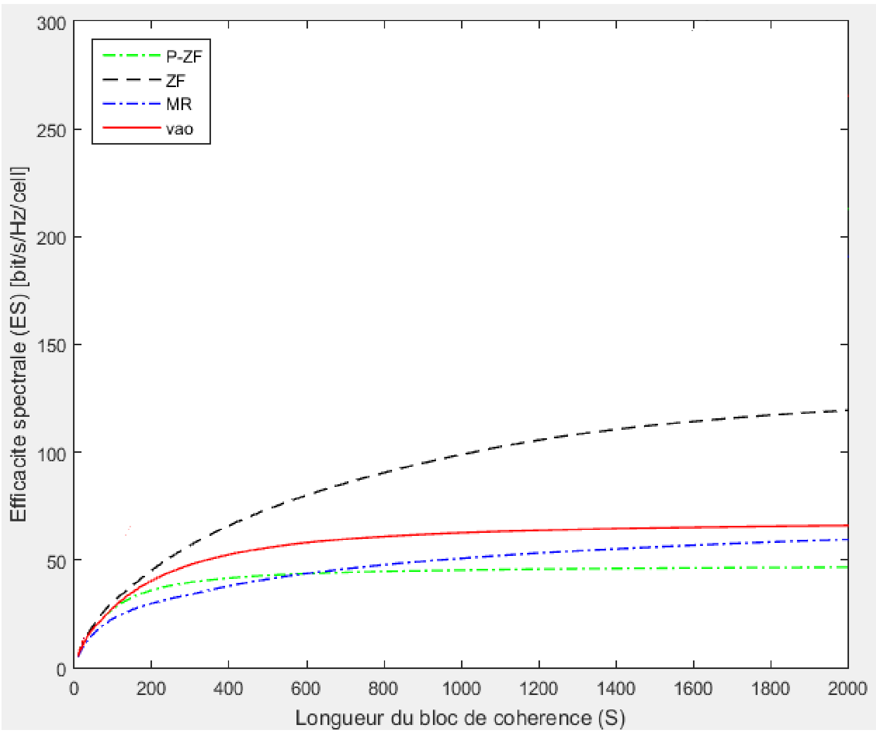

Fig 9:SE depending of S for M=100

In the case of M = 100 antennas, the increase gain of S greater than 500 is relatively small. In the case of M = 500, the system can use a growing S to plan more UEs and make major improvements to the ES. As the number of UEs increases, the part of the intracellular interference that can not be rejected due to the imperfection of the CSI becomes the main limiting factor. The advantage of P-ZF decreases. ZF and VAO keep their performance.

4. Conclusion

P-ZF is the most efficient in terms of spectral efficiency per cell when M is very widebut for M ranging from 10 to 500, VAO is the most efficient. MR is the worst performer foranyvalue of M. With VAO, the spectral efficiency achievable can go up to180bit / s / Hz / cell.VAO plans a large number of UEs, this is thanks to the fact that it canremove interference between UEs, 180 simultaneous communications are allowed.The SE per UE is 1 to 2.5 bit / s / Hz. P-ZF only suppresses interference butdoes not increase the level of the desired signal, it is very sensitive to the channel estimation error andloses its performance when the number of users in the cell is very high. In that case,VAO is the best choice since it seeks a balance between increasing the level ofuseful signal and suppression of interference. High spectral efficiency does not implya larger number of scheduled UEs, the optimal number varies withscheme and depends mainly on the number of BS antennas. For any coding used, an SNR of approximately5dB allows the system to keep its performance. ES depends more on the number of base station antennas than the length of the coherence block.

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