LOGNORMAL DISTRIBUTION OF DEMAND FOR EQUITY SHARES

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Abstract

In this paper we show that the demand dynamics of equity is a random walk process. This property enables us to model the distribution of the process of demand for equity by lognormal distribution.

Key words: Lognormal distribution, Skewness, Random Walk, Equity and Demand

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Introduction:

Lognormal distribution is important in modeling naturally occurring variables which are a product of a number of other naturally occurring variables. An example like the volume of gas in petroleum is often log normally distributed because it is a product of area of formation, its thickness, formation pressure, porosity and the gas-liquid ratio, Vosesoftware (2007). Demand for equity is also a natural phenomenon which depends on other natural responses like price of equity, purchasing power of buyers, market type etc.

Lognormal distribution is an application of a normal distribution rather than a separate distribution itself. We normally use it in naturally occurring events that are skewed when the natural logarithms of variables are normally distributed (op sit). Many measurements appear to be less or more skewed distributions and are usually common when the mean values of variables are low whereas their variances are large and none of the values takes a negative value. Skewed distributions thus often closely fit the lognormal distribution, Aitchison et al (1957); Shimizu et al (1980); Lee (1992); Johnson et al (1994); Sachs (1997); Eckhard (2001).

When the quantity of demand taken over a given period of time is transformed by taking the natural logarithms of each quantity of demand, the transformed demand becomes log normally distributed. Equity prices have always been modeled by lognormal distribution because prices are naturally occurring variables. Equity prices usually depend on;

(a) The original stock and the rate of return for an interval of time.

(b) The rate of return which follows a normal distribution but since normal distributions accommodate negative values, it is not possible with price values.

(c) The time interval for which the return is compounded. Compounding of return over a given time period is an important concept in finance, Waqqas farooq.com (2007).

According to Egan(2007), normal distribution and its "cousin" lognormal distribution are the two most commonly used distributions in the analysis of financial returns and prices and that in practice the lognormal distribution has been found to be a usefully accurate description of the distribution of the prices for many financial assets. He concludes his observation by submitting that normal distributions are often good in the approximation for returns.

The application of lognormal distribution has been used in areas like recreation, Englin et al (2011), stock prices and return, Insurance, Anssi et al(2011). The latter submitted that the shape of the demand distribution can differ considerably from one situation to another in that it can exhibit a strong positive skewness if there is a significant upside potential in demand or may have two peaks if there is a possibility of the introduction of a substitute product into the market.

In this paper we develop the lognormal distribution of the demand for equity shares assuming that there is no arrival of substitute equity to influence the existing market situation so as to change the trend of the usual demand dynamics. We thus assume that the natural logarithm of the quantity of equity demanded is skewed. Using a simple and straightforward methodology, we develop the lognormal distribution of the demand for equity without the use of a complicated mathematical approach.

The remaining part of this paper is divided into section 2 where we develop the distribution of the quantity of equity demanded and section 3 where we make our conclusion.

2. The distribution of the demand of equity shares

In this section, we discuss the distribution of the demand dynamics of capital shares traded by assuming that at the beginning of trade, the original quantity of a particular equity demanded is denoted by, $Q(0)$ and that this is done at time $t = 0$. We also assume that a proportional small change of δ is realized between the present quantity of demand and the next. This small change occurs after every unit change of time, which is in seconds or hours or days etc.

We therefore mean that the quantity of demand when $t = 1$ is given by;

$Q(1) = Q(0) + \delta Q(0)$

Or $Q(1) = Q(0)[1 + \delta]$, while at $t = 2$ the demanded quantity is given by;

$Q(2) = Q(1)[1 + \delta]$

And similarly at $t = 3$; $Q(3) = Q(2)[1 + \delta]$

Assuming this trend, we postulate that the amount of quantity of equity demanded at time t is given by the following relationship;

$$Q(t) = Q(t-1)[1 = \delta] \qquad (2.1)$$

Or $$\frac{Q(t)}{Q(t-1)} = 1 + \delta \qquad (2.2)$$

Taking the natural logarithm on both sides of equation (2.2), we get

$$ln\left(\frac{Q(t)}{Q(t-1)}\right) = \delta \qquad (2.3)$$

Where for all small δ, $ln(1+\delta) = \delta$.

Up to this stage, we have assumed that the small change δ is constant throughout each unit time intervals. This assumption is ideally not possible for the quantity of equity demanded which decrease or increase disproportionately. We therefore now add a subscript t to the small change to reflect this variation. We now write equation (2.3) as follows;

$$ln\left(\frac{Q(t)}{Q(t-1)}\right) = \delta_t \qquad (2.4)$$

Or $Q(t) = Q(t-1)[1+\delta]$ (2.5)

This follows accordingly that, at time,

$t = 1, Q(1) = Q(0)[1+\delta_1]$ or $\frac{Q(1)}{Q(0)} = 1+\delta_1$

$t = 2, Q(2) = Q(1)[1+\delta_2]$ or $\frac{Q(2)}{Q(1)} = 1+\delta_2$, thus

$$\frac{Q(2)}{Q(0)} = [1+\delta_1][1+\delta_2]$$

Proceeding this way, we postulate the quantity equity demanded in the interval $t = 0$ and $t = T-1$ to be given by the following relationship;

$$\frac{Q(T-1)}{Q(0)} = (1+\delta_1)(1+\delta_2)\ldots.(1+\delta_{T-1}) \qquad (2.6)$$

Or $ln\left(\frac{Q(T-1)}{Q(0)}\right) = \delta_1 + \delta_2 + \cdots \delta_{T-1}$ (2.7)

$$= \sum_{i=1}^{T-1} \delta_t$$

And similarly the capital shares demanded in the time interval $t = 0$ and $t = T$ is given by;

$$\frac{Q(T)}{Q(0)} = (1+\delta_1)(1+\delta_2)\ldots.(1+\delta_T) \qquad (2.8)$$

Or $ln\left(\frac{Q(T)}{Q(0)}\right) = \delta_1 + \delta_2 + \cdots \delta_T$ (2.9)

$$= \sum_{i=1}^{T} \delta_t$$

Subtracting equation (2.7) from Equation (2.9) gives;

$$\ln Q(T) - lnQ(T-1) = \delta_t \qquad (2.10)$$

Or $lnQ(T) = lnQ(T-1) + \delta_t$ (2.11)

Equation (2.11) is in the form of $q_t = q_{t-1} + \epsilon_t$ which is a random walk property. Liang (2003) submits that a random walk property enables a process to be used in the modeling of the dynamics of a capital market hence this qualifies the quantity of demanded equity being discussed here as a candidate.

At this stage, we now examine the distribution of the quantity of equity demanded at time t .

Equation (2.9) can be rewritten in the following form;

$$lnQ(T) = lnQ(0) + \sum_{i=1}^{T} \delta_t \qquad (2.12)$$

Where the δ_t's are normally distributed with $E(\delta_t) = \mu$ and $Var(\delta_t) = \sigma^2$ meaning that $lnQ(T)$ is also normally distributed with $E\big(lnQ(T)\big) = lnQ(0) + \mu T$ and $Var\big(lnQ(T)\big) = \sigma^2 T$. That is; $lnQ(T) \sim N(lnQ(0) + \mu T, \sigma^2 T)$. Taking the interval of time, $t \in [0, T]$, we have

$$lnQ(T) = lnQ(0) + \mu T + \sigma T \qquad (2.13)$$

Or $$Q(T) = Q(0)exp(\mu T + \sigma T) \qquad (2.14)$$

This means that $E(Q(T)) = Q(0)exp(\mu T)$ and it can also be shown that $Var(Q(T)) = Q^2(0)exp(\mu T)[exp(\sigma^2 T - 1)]$ Taking $t = T$ to be our time in consideration, we therefore conclude that, $Q(t)$ is log normally distributed, that is $Q(t) \sim lognormal\big(Q(0)exp(\mu T), Q(0)exp(2\mu T)(exp(\sigma^2 T - 1)\big)$.

3.0 Conclusion

Trade in equity has become attractive to traders as economies of nations get better. This has enabled investors to become more willing to put some money in equities to achieve higher rates of return. This trend in growth for equity demand has been realized both in developed and developing economies. In this paper, we have shown that this growth of demand is a random walk process. We have also used a simple mathematical approach to show that this process is log normally distributed.

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