OPTIMUM COST- TIME TRADE OFF IN A CAPACITATED FIXED CHARGE TRANSPORTATION PROBLEM WITH BOUNDS ON RIM CONDITIONS

KAVITA GUPTA*

S.R. ARORA**

Abstract:

In this paper, an algorithm is presented to find the optimum time cost trade off in a capacitated fixed charge transportation problem giving the same priority to both time and cost. Sometimes, there is a condition that we can send an amount more than or less than a certain specified amount which gives rise to capacitated time minimizing transportation problem. Moreover, sometimes a fixed cost (like set up cost for machines, landing fees at an airport, cost of renting a vehicle) is also associated with every origin that gives rise to fixed charge problems. From the practical point of view, the cost minimizing transportation problem and the time minimizing transportation problem can not be viewed as two independent problems. In this paper, an algorithm is presented that gives efficient time cost trade off pairs which minimizes cost and time simultaneously in a capacitated fixed charge transportation problem. A numerical example is given to illustrate the developed algorithm.

Keywords: transportation problem, trade off, optimum time cost trade off , capacitated transportation problem, fixed charge transportation problem.

---

* Department of Mathematics, Jagan Institute of Management Studies, 3 Institutional Area, Sector-5, Rohini , Delhi, India

** Ex-Principal, Hans Raj College, University of Delhi, Delhi-110007, India.

1 Introduction

The fixed charge transportation problem was originally formulated by Dantzig and Hirisch [10] in 1954.Then Murthy [11] solved the fixed charge problem by ranking the extreme points . In real world situations, when a commodity is transported ,a fixed cost is incurred in the objective function .The fixed cost may represent the cost of renting a vehicle, landing fees at an airport, set up cost for machines etc. Sandrock [14] discussed fixed charge transportation problem in 1982.

Sometimes, there may exist emergency situations such as fire services, ambulance services, police services etc when the time of transportation is more important than cost of transportation. Arora and Ahuja [1]; Garfinkel and Rao [8] and Hammer [9] have studied the time minimizing transportation problem which is a special case of bottleneck linear programming problems. Pandian and Natarajan [12-13] gave a new method namely , Blocking method for finding an optimal solution to bottleneck transportation problem. Moreover Sharma et.al.[15-16] studied a capacitated two stage time minimization transportation problem. If the total flow in a transportation problem with bounds on rim conditions is also specified, the resulting problem makes the transportation problem more realistic. Moreover, if the total capacity of each route is also specified then optimal solution of such problems is of greater importance which gives rise to capacitated transportation problems. Capacitated transportation problems have been studied by various authors. Dahiya and Verma [7] discussed capacitated transportation problems with bounds on rim conditions.Arora and Gupta[2-5] have contributed a lot in the field of capacitated transportation problem. Basu, Pal and Kundu [6] developed an algorithm for the optimum time cost trade off in a fixed charge linear transportation problem giving same priority to cost and time.

In this paper, we have unified the two objectives of minimizing cost and time in a capacitated fixed charge transportation problems with bounds on rim conditions.

2 Problem Formulation:

The general model of the capacitated fixed charge transportation problems with bounds on rim conditions is given below:

$$(P1): \min\{ \sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij} + \sum_{i\in I} F_i, \max_{i\in I, j\in J}(t_{ij} / x_{ij} > 0) \}$$

subject to

$$a_i \le \sum_{i\in I} x_{ij} \le A_i \qquad \forall\, i \in I \qquad (1.1)$$

$$b_j \le \sum_{j\in J} x_{ij} \le B_j \qquad \forall\, j \in J \qquad (1.2)$$

$$l_{ij} \le x_{ij} \le u_{ij} \text{ and integers } \forall\, i \in I,\, j \in J \qquad (1.3)$$

$I = \{1, 2, \ldots m\}$ is the index set of m origins.

$J = \{1, 2, \ldots, n\}$ is the index set of n destinations.

x_{ij} = number of units transported from i^{th} origin to j^{th} destination .

c_{ij} = cost of transporting one unit of commodity from i^{th} origin to j^{th} destination.

l_{ij} and u_{ij} are the bounds on number of units to be transported from i^{th} origin to j^{th} destination.

a_i and A_i are the bounds on the availability at the i^{th} origin, $i \in I$

b_j and B_j are the bounds on the demand at the j^{th} destination, $j \in J$

t_{ij} is the time of transporting goods from i^{th} origin to the j^{th} destination.

F_i is the fixed cost associated with i^{th} origin.

For the formulation of F_i (i=1,2 … m), we assume that F_i (i = 1, 2 .. m) has p number of steps so that

$$F_i = \sum_{l=1}^{p} F_{il}\delta_{il} \qquad , i=1, 2, 3\ldots m \quad , l=1,2,3\ldots p$$

$$\text{where } \delta_{il} = \begin{cases} 1 & \text{if } \sum_{j=1}^{n} x_{ij} > a_{il} \\ 0 & \text{otherwise} \end{cases} \quad \text{for } l=1,2,3\ldots\ldots p,\ i=1,2,\ldots\ldots m$$

Here, $0 = a_{i1} < a_{i2} \dots < a_{ip.}$ $a_{i1}, a_{i2} \dots, a_{ip}$ $(i = 1,2, \dots m)$ are constants and F_{il} are the fixed costs. $\forall$ i= 1, 2 …m, and l = 1,2 ..p

The problem (P1) is solved in the following way.

(1) First, we minimize cost without considering time and then minimize time with respect to the minimum cost obtained.

(2) Secondly, after defining a new cost as follows with respect to minimum time obtained in the last result, we minimize cost. Then we minimize time with respect to the minimum cost of last result. Step (2) is repeated until the solution is infeasible. This is known as re-optimisation procedure.

$$c^1_{ij} = \begin{Bmatrix} M & \text{if } t_{ij} \ge T^1 \\ c_{ij} & \text{if } t_{ij} < T^1 \end{Bmatrix}$$

The above problem (P1) is separated in to two problems (P2) and (P3) for solving it by re-optimisation procedure, where

(P2):$\min(\sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{i \in I} F_i)$ subject to (1.1) ,(1.2) and (1.3) and

(P3): $\max \; t_{ij} / x_{ij} > 0$ $\forall$ i= 1, 2 …m and j=1,2,….n subject to (1.1) ,(1.2) and (1.3)

To solve the problem (P2), we first convert it in to related problem (P2´) given below.

(P2´): $\min(\sum_{i \in I'} \sum_{j \in J'} c_{ij}' y_{ij} + \sum_{i \in I'} F_i')$ subject to

$$\sum_{j \in J'} y_{ij} = A_i' \quad \forall i \in I'$$

$$\sum_{i \in I'} y_{ij} = Bj' \quad \forall j \in J'$$

$$l_{ij} \le y_{ij} \le u_{ij} \quad \forall i \in I, j \in J$$

$$0 \le y_{m+1,j} \le B_j - b_j \quad \forall j \in J$$

$0 \le y_{i,n+1} \le A_i - a_i \quad \forall i \in I$

$y_{m+1,n+1} \ge 0$ and integers

where $A_i' = A_i \ \forall i \in I$, $\quad A'_{m+1} = \sum_{j \in J} B_j$, $B'_j = B_j \ \forall j \in J$, $B'_{n+1} = \sum_{i \in I} A_i$

$c'_{ij} = c_{ij}$, $\forall i \in I, j \in J$, $\quad c'_{m+1,j} = c'_{i,n+1} = c'_{m+1,n+1} = 0 \quad \forall i \in I, \quad \forall j \in J$

$F_i' = F_i \quad \forall i = 1,2 \ldots m, \ F'_{m+1} = 0$

$I' = \{1, 2, \ldots m, m+1\}, \ J' = \{1, 2, \ldots n, n+1\}$

To solve the problem (P3), we convert it in to related problem (P3´) given below.

(P3´): min $T = \max \ t'_{ij} / x'_{ij} > 0 \quad \forall i \in I', j \in J'$

subject to

$$\sum_{j \in J'} x'_{ij} = A_i' \quad \forall i \in I'$$

$$\sum_{i \in I'} x'_{ij} = B_j' \quad \forall j \in J'$$

$l_{ij} \le x'_{ij} \le u_{ij} \quad \forall i \in I'$ and $\forall j \in J'$

$0 \le x'_{m+1,j} \le B_j - b_j \quad \forall j \in J$

$0 \le x'_{i,n+1} \le A_i - a_i \quad \forall i \in I$

$x_{m+1,n+1} \ge 0$ and integers

$A_i' = A_i \ \forall i \in I$, $\quad A'_{m+1} = \sum_{j \in J} B_j$, $B'_j = B_j \ \forall j \in J$, $B'_{n+1} = \sum_{i \in I} A_i$,

$t'_{ij} = t_{ij}$, $\forall i \in I, j \in J$, $\quad t'_{m+1,j} = t'_{i,n+1} = t'_{m+1,n+1} = 0$, $x'_{ij} = x_{ij}$, $\forall i \in I$, $\forall j \in J$

$I' = \{1, 2, \ldots m, m+1\}, \ J' = \{1, 2, \ldots n, n+1\}$

To obtain the set of efficient time cost trade off pairs, we first solve (P2´) and read the time with respect to the minimum cost Z where time T is given by problem (P3´).At the first iteration, let Z_1* be the minimum total cost of the problem (P2´) Find all alternate solutions i.e. solutions having the same value of Z = Z_1*. Let these solutions be $X_1, X_2, \ldots X_n$. Corresponding to these solutions, find the time $T_1^* = \min_{X_1, X_2 \ldots X_n} \max_{i \in I, j \in J'} t_{ij} / x_{ij} > 0$ .Then

(Z_1^*, T_1^*) is called the first cost time trade off pair. Modify the cost with respect to the time so obtained i.e. define $c_{ij} = \begin{cases} M & \text{if } t_{ij} \geq T^* \\ c_{ij} & \text{if } t_{ij} < T^* \end{cases}$ and form the new problem and find its optimal solution and all feasible alternate solutions. Let the new value of Z be Z_2* and the corresponding time is T_2* , then (Z_2*, T_2*) is the second cost time trade off pair. Repeat this process. Suppose that after q th iteration ,the problem becomes infeasible. Thus, we get the following complete set of cost- time trade off pairs. (Z_1*, T_1*) , (Z_2*, T_2*),(Z_3*,T_3*),……………(Z_q*, T_q*) where $Z_1^* \leq Z_2^* \leq Z_3^* \leq \ldots \leq Z_q^*$ and $T_1^* > T_2^* > T_3^* \ldots > T_q^*$.The pairs so obtained are pareto-optimal solution of the given problem. Then we identify the minimum cost Z_1* and minimum time T_q* among the above trade off pairs. The pair (Z_1*, T_q*) with minimum cost and minimum time is termed as the ideal pair which can not be achieved in practical situations.

3 Theoretical development:

Theorem 1: There is one to one correspondence between a feasible solution of problem (P2) and a feasible solution of problem (P2´).

Proof: Let $\{y_{ij}\}_{I' x J'}$ be a feasible solution of the problem (P2´) . Define $\{x_{ij}\}$, $i \in I$, $j \in J$ by the following transformation

$$x_{ij} = y_{ij} \quad \forall\, i \in I, j \in J \tag{1.4}$$

We will prove that $\{x_{ij}\}$ is a feasible solution of the problem (P_2)

Since $\{y_{ij}\}$ is the feasible solution of problem (P2´)

$$\sum_{j \in J'} y_{ij} = A_i' \;, \forall i \in I' \tag{1.5}$$

$$\sum_{i\in I'} y_{ij} = B'_j \ , \ \forall j \in J' \qquad (1.6)$$

$$l_{ij} \le y_{ij} \le u_{ij}; \forall i \in I', \forall j \in J' \qquad (1.7)$$

$$0 \le y_{i,n+1} \le A_i - a_i ; \forall i \in I \text{ and } 0 \le y_{m+1,j} \le B_j - b_j \ ; \ \forall j \in J \qquad (1.8)$$

By relation (1.4) and (1.7) we get

$l_{ij} \le x_{ij} \le u_{ij} \ \forall i\in I, j\in J$ as $I \subset I'$ and $J \subset J'$

$\sum_{j\in J'} y_{ij} = A'_i \ \ \forall i \in I'$ By (1.5)

$\sum_{j\in J} y_{ij} + y_{i,n+1} = A_i \ ; \forall i \in I$ (since $A_i' = A_i \ \forall i\in I \subset I'$)

$\sum_{j\in J} y_{ij} = A_i - y_{i,n+1} \ ; \forall i \in I$

By (1.8),we have $0 \le y_{i,n+1} \le A_i - a_i ; \forall i \in I$

or $a_i \le A_i - y_{i,n+1} \le A_i ; \forall i \in I$

$\Rightarrow a_i \le \sum_{j\in J} y_{ij} \le A_i ; \forall i \in I$

$\Rightarrow a_i \le \sum_{j\in J} x_{ij} \le A_i \ ; \forall i \in I$ by(1.4)

Similarly, $b_j \le \sum_{i\in I} x_{ij} \le B_j ; \forall j \in J$

$\therefore \{x_{ij}\}, \ \forall i \in I, \forall j \in J$ is the feasible solution of problem (P2)

Conversely, let $\{x_{ij}\}, \ i\in I, \ j\in J$ be a feasible solution of problem (P2).

We will show that $\{y_{ij}\}$ $i\in I'$, $j\in J'$ is a feasible solution of problem (P2´) where $\{y_{ij}\}$ is defined as follows:

$$y_{ij} = x_{ij}, \forall i \in I, j \in J \quad (1.9)$$

$$y_{i,n+1} = A_i - \sum_{j \in J} x_{ij} \;;\forall i \in I \quad (1.10)$$

$$y_{m+1,j} = B_j - \sum_{i \in I} x_{ij}; \forall j \in J \quad (1.11)$$

$$y_{m+1,n+1} = \sum_{i \in I} \sum_{j \in J} x_{ij} \quad (1.12)$$

Since $\{x_{ij}\}$, $i \in I$, $j \in J$ is the feasible solution of problem (P2)

$$\therefore\; l_{ij} \le x_{ij} \le u_{ij} \quad \forall\, i \in I, j \in J$$

$$\Rightarrow l_{ij} \le y_{ij} \le u_{ij} \quad \forall\, i \in I, j \in J \qquad \text{By } (1.9)$$

Also, $a_i \le \sum_{j \in J} x_{ij} \le A_i \quad ; \forall i \in I$

or $0 \le A_i - \sum_{j \in J} x_{ij} \le A_i - a_i \;; \quad \forall i \in I$

$$\Rightarrow 0 \le y_{i,n+1} \le A_i - a_i \quad ; \forall i \in I \qquad \text{By } (1.10)$$

$$\Rightarrow\; l_{i,n+1} \le y_{i,n+1} \le u_{i,n+1}; \quad \forall i \in I$$

Also, $b_j \le \sum_{i \in I} x_{ij} \le B_j; \; \forall j \in J$

or $0 \le B_j - \sum_{i \in I} x_{ij} \le B_j - b_j \;; \; \forall j \in J$

$$\Rightarrow\; 0 \le y_{m+1,j} \le B_j - b_j; \forall j \in J \qquad \text{By } (1.11)$$

$$\Rightarrow l_{m+1,j} \le y_{m+1,j} \le u_{m+1,j} \quad ; \forall j \in J$$

Clearly, $0 \le y_{m+1,n+1} = \sum_{i \in I} \sum_{j \in J} x_{ij} \le M$

$$\therefore\quad l_{ij} \le y_{ij} \le u_{ij}; \forall i \in I', j \in J'$$

Now, $\sum_{j\in J'} y_{ij} = \sum_{j\in J} y_{ij} + y_{i,n+1}$

$= \sum_{j\in J} x_{ij} + A_i - \sum_{j\in J} x_{ij}$ By (1.9) and (1.10)

$= A_i \;\; ; \forall i \in I$

$= A'_i \;\; ; \forall i \in I$

Moreover, $\sum_{i\in I'} y_{ij} = \sum_{i\in I} y_{ij} + y_{m+1,j}$

By(1.6) and (1.11)

$= \sum_{i\in I} x_{ij} + B_j - \sum_{i\in I} x_{ij} \ldots$

$= B_j \;\; ; \forall j \in J$

$= B_j' \;\; ; \forall j \in J$

Now, we will show that $A'_{m+1} = \sum_{j\in J'} y_{m+1,j}$

For i = m+1,

$\sum_{j\in J'} y_{m+1,j} = \sum_{j\in J} y_{m+1,j} + y_{m+1,n+1}$

$= \sum_{j\in J}\left(B_j - \sum_{i\in I} x_{ij}\right) + \sum_{i\in I}\sum_{j\in J} x_{ij}$ By (1.11) and (1.12)

$= \sum_{j\in J} B_j$

$= A'_{m+1}$

Similarly , $B'_{n+1} = \sum_{i\in I'} y_{i,n+1}$

$\therefore$ $\{y_{ij}\}$ is a feasible solution of problem (P2´)

Theorem 2: The value of objective function of problem (P2´) at a feasible solution is equal to value of objective function of problem (P_2) at its corresponding feasible solution and conversely.

Proof: Let $\{y_{ij}\}_{I'xJ'}$ and $\{x_{ij}\}_{IxJ}$ be corresponding feasible solution of problem (P2´) and problem(P2)respectively.

Then Z = objective function value of (P2´) at $\{y_{ij}\}$

$$= \sum_{i\in I'} \sum_{j\in J'} c'_{ij} y_{ij} + \sum_{i\in I'} F_i^{'}$$

$$= \sum_{i\in I} \sum_{j\in J} c'_{ij} y_{ij} + \sum_{i\in I} c'_{i,n+1,}\ y_{i,n+1} + \sum_{j\in J} c'_{m+1,j}\ y_{m+1,j} + c'_{m+1,n+1}\ \ y_{m+1,n+1} + \sum_{i\in I} F_i' + F'_{m+1}$$

$$= \sum_{i\in I} \sum_{j\in J} c_{ij} x_{ij} + \sum_{i\in I} F_i \text{ because } \left\{ \begin{array}{l} c_{ij}^{'} = c_{ij}, \forall i \in I, j \in J \\ x_{ij} = y_{ij}, \forall i \in I, j \in J \\ c_{i,n+1}^{'} = c_{m+1,j}^{'} = c_{m+1,n+1}^{'} = 0 \\ F_{m+1}^{'} = 0, F_i^{'} = F_i, \forall i \in I \end{array} \right\}$$

= objective function value of problem (P_2) at $\{x_{ij}\}$

Converse can be proved in a similar way

Theorem 3: There is a one to one correspondence between the optimal solution to problem (P2´) and the optimal solution to problem (P_2)

Proof: Let $\{\hat{x}_{ij}\}_{IxJ}$ be an optimal solution to Problem (P2) with the value of objective function as Z^0. Since $\{\hat{x}_{ij}\}_{IxJ}$ is an optimal solution, ∴ $\{x_{ij}\}$ is a feasible solution to problem (P2). Then by theorem 1, there exist a corresponding feasible solution $\{\hat{y}_{ij}\}_{I'xJ'}$ to problem (P2´). The value yielded by $\{\hat{y}_{ij}\}$ is Z^0 [refer to theorem 2].

Now we will show that $\{\hat{y}_{ij}\}_{I'xJ'}$ is the optimal solution to problem (P2´).

Let if possible, $\{\hat{y}_{ij}\}$ be not an optimal solution to problem (P2´). ∴ there exist a feasible solution $\{y'_{ij}\}$ say to problem (P2´) having the value of objective function $Z' < Z^0$. Let $\{x'_{ij}\}$ be the corresponding feasible solution to problem (P2). Then by theorem 2,

$$Z' = \sum_{i \in I} \sum_{j \in J} c_{ij} x'_{ij} + \sum_{i \in I} F_i < Z^0$$

which contradicts that $\{\hat{x}_{ij}\}$ is an optimal solution to problem (P2).

Similarly, starting from an optimal feasible solution to problem (P2´), one can derive an optimal feasible solution to problem (P2) having the same objective function value.

Theorem 4: Let X = $\{X_{ij}\}$ be a basic feasible solution of problem (P2) with basis matrix B. Then it will be an optimal basic feasible solution if

$$R^1_{ij} = \theta_{ij} \;\; c_{ij} - z_{ij} \;\; + \Delta F_{ij} \geq 0; \forall (i, j) \in N_1$$

and $R^2_{ij} = -\theta_{ij} \;\; c_{ij} - z_{ij} \;\; + \Delta F_{ij} \geq 0; \forall (i, j) \in N_2$

such that

$$u_i + v_j = c_{ij} \qquad \forall (i, j) \in B$$

$$u_i + v_j = z_{ij} \qquad \forall (i, j) \in N_1 \text{ and } N_2$$

ΔF_{ij} is the change in fixed cost $\sum_{i \in I} F_i$ when some non basic variable x_{ij} undergoes change by an amount of θ_{ij}.

θ_{ij} = level at which a non basic cell (i,j) enters the basis replacing some basic cell of B.

N_1 and N_2 denotes the set of non basic cells (i,j) which are at their lower bounds and upper bounds respectively.

Note: u_i, v_j are the dual variables which are determined by using above equations and taking one of the u_i's or v_j's. as zero.

Proof: Let Z^0 be the objective function value of the problem (P2).

Let $z^0 = Z_1 + F^0$ where $F^0 = \sum_{i \in I} F_i$ and $Z_1 = \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij}$

Let $\hat{z}$ be the objective function value at the current basic feasible solution $\hat{X} = \{x_{ij}\}$ corresponding to the basis B obtained on entering the non basic cell $x_{ij} \in N_1$ in to the basis which undergoes change by an amount θ_{ij} and is given by min{$u_{ij} - l_{ij}$; x_{ij} - l_{ij} for all basic cells (i,j) with a (- θ)entry in the θ-loop; $u_{ij} - x_{ij}$ for all basic cells (i,j) with a (+ θ)entry in the θ-loop}.

Then $\hat{z} = \left[z_1 + \theta_{ij}\left(c_{ij} - z_{ij}\right)\right] + F^0 + \Delta F_{ij}$

$$\hat{z} - z^0 = \theta_{ij}(c_{ij} - z_{ij}) + \Delta F_{ij}$$

This basic feasible solution will give an improved value of z if $\hat{z} < z^0$ .It means

If $\theta_{ij}(c_{ij} - z_{ij}) + \Delta F_{ij} < 0$ (1.13)

Therefore one can move from one basic feasible solution to another basic feasible solution on entering the cell (i,j) $\in$ N_1 in to the basis for which condition (1.13) is satisfied.

It will be an optimal basic feasible solution if

$$R^1_{ij} = \theta_{ij}\ c_{ij} - z_{ij}\ + \Delta F_{ij} \geq 0; \forall (i,j) \in N_1$$

Similarly, when non basic variable $x_{ij} \in N_2$ undergoes change by an amount θ_{ij} then

$$\hat{z} - z^0 = -\theta_{ij}(c_{ij} - z_{ij}) + \Delta F_{ij} < 0$$

It will be an optimal basic feasible solution if

$$R^2_{ij} = -\theta_{ij}\ c_{ij} - z_{ij}\ + \Delta F_{ij} \geq 0; \forall (i,j) \in N_2$$

4 ALGORITHM

Step1. Starting from the given problem (P1), separate it in to two problems (P2) and (P3). Form the related problem (P2´).Find an initial basic feasible solution to the problem (P2´) with respect to the variable costs by upper bound simplex technique . Let B be the current basis.

Step 2. Calculate the fixed cost of the current basic feasible solution and denote it by F(current) where F(current) = $\sum_{i=1}^{m} F_i$

Step 3(a). Find $\Delta F_{ij} = F(NB) - F(current)$ where F(NB) is the total fixed cost obtained when some non basic cell (i,j) undergoes change.

Step 3(b): Calculate θ_{ij} ,(c_{ij}-z_{ij}) for all non basic cells such that

$$u_i + v_j = c_{ij} \quad \forall (i, j) \in B$$

$$u_i + v_j = z_{ij} \quad \forall (i, j) \in N_1 \text{ and } N_2$$

θ_{ij} = level at which a non basic cell (i,j) enters the basis replacing some basic cell of B.

N_1 and N_2 denotes the set of non basic cells (i,j) which are at their lower bounds and upper bounds respectively.

Note: u_i, v_j are the dual variables which are determined by using above equations and taking one of the u_i's or v_j's. as zero.

Step 3(c) : Find $R^1_{ij}; \forall (i, j) \in N_1$ and $R^2_{ij}; \forall (i, j) \in N_2$ where

$$R^1_{ij} = \theta_{ij}\ c_{ij} - z_{ij}\ + \Delta F_{ij} \ge 0; \forall (i, j) \in N_1 \text{ and } R^2_{ij} = -\theta_{ij}\ c_{ij} - z_{ij}\ + \Delta F_{ij} \ge 0; \forall (i, j) \in N_2$$

Step 4: If $R^1_{ij} \ge 0; \forall (i, j) \in N_1$ and $R^2_{ij} \ge 0; \forall (i, j) \in N_2$ then the current solution so obtained is the optimal solution to (P2´).Go to step 5.Otherwise, some (i,j) $\in N_1$ for which $R^1_{ij} < 0$ or some (i,j) $\in N_2$ for which $R^2_{ij} < 0$ will undergo change. Go to step 2.

Step 5: Let Z^1 be the optimal cost of (P2´) yielded by the basic feasible solution {y'_{ij}}. Find all alternate solutions to the problem (P2´) with the same value of the objective function. Let these solutions be $X_1, X_2, \ldots, X_n$ and $T_1 = \min_{X_1, X_2 \ldots X_n} \max_{i \in I, j \in J'} t_{ij} / x_{ij} > 0$ .Then the corresponding pair (Z^1, T^1) will be the first time cost trade off pair for the problem (P1).To find the second cost- time trade off pair, go to step 6.

Step6: Define $c^1_{ij} = \begin{Bmatrix} M & \text{if } t_{ij} \geq T^1 \\ c_{ij} & \text{if } t_{ij} < T^1 \end{Bmatrix}$

where M is a sufficiently large positive number. Form the corresponding capacitated fixed charge transportation problem with variable cost c^1_{ij}. Repeat the above process till the problem becomes infeasible. The complete set of time cost trade off pairs of (P1) at the end of q^{th} iteration are given by $(Z^1, T^1), (Z^2,T^2)\ldots\ldots\ldots(Z^q, T^q)$ where $Z^1 \leq Z^2 \leq \ldots\ldots \leq Z^q$ and $T^1 > T^2 > \ldots\ldots > T^q$.

Remark 1: The pair (Z^1, T^q) with minimum cost and minimum time is the ideal pair which can not be achieved in practice except in some trivial case.

Convergence of the algorithm: The algorithm will converge after a finite number of steps because we are moving from one extreme point to another extreme point and the problem becomes infeasible after a finite number of steps.

5.Numerical Illustration:

Consider the following 2 x 3 capacitated fixed charge transportation problem with bounds on rim conditions. Table 1 gives the values of c_{ij}, A_i ,B_j for i=1,2 and j=1,2,3.Table 2 gives values of t_{ij} for i =1,2 and j =1,2,3

Table 1:cost matrix of problem (P1)

	D_1	D_2	D_3	A_i
O_1	5	9	9	30
O_2	4	6	2	40
B_j	30	20	30	

Table 2 :Time matrix of problem (P1)

	D_1	D_2	D_3
O_1	15	8	13

O_2	10	13	11

Note:O_1 and O_2 are origins.D_1,D_2,D_3 are the destinations . c_{ij} is the cost mentioned in table 1 at the upper left corner of each cell and t_{ij} is the time in table 2.

$$5 \le \sum_{j=1}^{3} x_{1j} \le 30 \;,\quad 10 \le \sum_{j=1}^{3} x_{2j} \le 40 \;,\; 10 \le \sum_{i=1}^{2} x_{i1} \le 30 \;,\quad 7 \le \sum_{i=1}^{2} x_{i2} \le 20 \;,\; 5 \le \sum_{i=1}^{2} x_{i3} \le 30$$

$1 \le x_{11} \le 10$, $2 \le x_{12} \le 10$, $0 \le x_{13} \le 5$,$0 \le x_{21} \le 15$, $3 \le x_{22} \le 15$, $1 \le x_{23} \le 20$

$F_{11} = 150$, $F_{12} = 50$, $F_{13} = 50$, $F_{21} = 200$, $F_{22} = 100$, $F_{23} = 50$

$$F_i = \sum_{l=1}^{2} F_{il}\delta_{il} \quad \text{for } l=1,2,3 \text{ where}$$

$$\delta_{i1} = \begin{cases} 1 & \text{if } \sum_{j=1}^{3} x_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_{i2} = \begin{cases} 1 & \text{if } \sum_{j=1}^{3} x_{ij} > 10 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_{i3} = \begin{cases} 1 & \text{if } \sum_{j=1}^{3} x_{ij} > 20 \\ 0 & \text{otherwise} \end{cases}$$

Introduce a dummy origin and a dummy destination in Table 1 with $c_{i4} = 0$ for all i = 1,2 and $c_{3j} = 0$ for all j = 1,2,3 Also we have $0 \le x_{14} \le 25$, $0 \le x_{24} \le 30$, $0 \le x_{31} \le 20$, $0 \le x_{32} \le 13$, $0 \le x_{33} \le 25$, $0 \le x_{34} \le M$ and $F_{3j} = 0$ for j=1,2,3,4 In this way , we form the problem (P2´).Similarly on introducing a dummy origin and a dummy destination in Table 2 with $t_{i4} = 0$ for i=1,2 and $t_{3j} = 0$ for j=1,2,3,4, we form problem (P3´) .Find an initial basic feasible solution of problem (P2´) which is given in table 3 below.

Table 3: A basic feasible solution of problem (P2´)

	D_1	D_2	D_3	D_4	u_i
O_1	5 **10**	9 $\underline{2}$	9	0 **18**	0
O_2	4 **0**	6 **5**	2 **5**	0 $\overline{30}$	-1
O_3	0 $\overline{20}$	0 $\overline{13}$	0 $\overline{25}$	0 **22**	0
v_j	5	7	3	0	

Note: Values in the upper left corner of each cell in table 3 are c_{ij} 's and entries of the form $\underline{a}$ and $\overline{b}$ in the upper right corner represent non basic cells which are at their lower bounds and upper bounds respectively. Entries in bold at the upper right corner represent basic cells.

F (current) = 200 + 200 + 0 = 400

Table 4: Optimality condition of problem (P2´)

NB	O_1D_2	O_1D_3	O_2D_4	O_3D_1	O_3D_2	O_3D_3
$(c_{ij}-z_{ij})$	2	6	1	-5	-7	-3
θ_{ij}	2	4	7	0	0	0
$\theta_{ij}(c_{ij}-z_{ij})$	4	24	7	0	0	0
F(NB)	400	400	450	400	400	400

ΔF_{ij}	0	0	50	0	0	0
R^1_{ij}	4	24				
R^2_{ij}			43	0	0	0

Since $R^1_{ij} \geq 0$; $\forall (i, j) \in N_1$ and $R^2_{ij} \geq 0; \forall (i, j) \in N_2$, the solution given in table 3 is an optimal solution of problem (P2´) and hence yields an optimal solution of (P2) with minimum cost Z^1 = 508 and the corresponding time T^1=15.Therefore the first time cost trade off pair is (508,15).

Define $c^1_{ij} = \begin{Bmatrix} M & \text{if } t_{ij} \geq 15 \\ c_{ij} & \text{if } t_{ij} < 15 \end{Bmatrix}$

A basic feasible solution to the new cost problem is given in table 5 below.

Table 5:A basic feasible solution to the new cost problem

	D_1	D_2	D_3	D_4	u_i
O_1	M $\underline{1}$	9 **4**	9	0 $\overline{25}$	3
O_2	4 **9**	6 **3**	2 **5**	0 **23**	0
O_3	0 $\overline{20}$	0 $\overline{13}$	0 $\overline{25}$	0 **22**	0
v_j	4	6	2	0	

F(current) = 150 + 300 + 0 = 450

Table 6: optimality condition of the new cost problem

NB	O_1D_3	O_1D_4	O_3D_1	O_3D_2	O_3D_3
$(c_{ij}-z_{ij})$	4	-3	-4	-6	-2
θ_{ij}	2	0	6	12	15
$\theta_{ij}(c_{ij}-z_{ij})$	8	0	-24	-72	-30
F(NB)	450	450	500	500	500
ΔF_{ij}	0	0	50	50	50
R^1_{ij}	8				
R^2_{ij}		0	74	122	80

Since $R^1_{ij} \geq 0$; $\forall(i, j) \in N_1$ and $R^2_{ij} \geq 0; \forall(i, j) \in N_2$,the solution given in table 5 is an optimal solution with minimum cost Z^2 =555 and the corresponding time T^2 =13.Therefore the second time cost trade off pair is (555,13).

Proceeding like this, the time cost trade off pairs are (508,15), (555,13), (555,11).If we proceed further, the problem becomes infeasible.

6 Conclusion

In this paper , we have proposed an algorithm to find optimum time – cost trade off pairs in a capacitated fixed charge transportation problem with bounds on total availabilities at sources and total destination requirements. We separated the problem in to two problems and formed the related fixed charge capacitated transportation problem by introducing a dummy source and a dummy destination to find the optimum time cost trade off pairs.

References

[1]Ahuja,A and Arora,S.R. , "A paradox in fixed charge transportation problem" , *Indian Journal Of Pure and Applied Mathematics*,31(7) (2000)809-822

[2]Arora, S. R and Gupta ,K ., " An algorithm for solving a capacitated fixed charge bi-criterion indefinite quadratic transportation problem with restricted flow" , *International Journal Of Research In IT, Management and Engineering (ISSN 2249-1619)*1(5) (2011) 123-140

[3]Arora, S.R and Gupta, K., "Restricted flow in a non linear capacitated transportation problem with bounds on rim conditions", *International Journal Of Management , IT and Engineering (ISSN- 2249-0558)2(5)* (2012)*226-243*

[4]Arora ,S.R and Gupta ,K., "An algorithm to find optimum cost time trade off pairs in a fractional capacitated transportation problem with restricted flow" *International Journal Of Research In Social Sciences (ISSN:2249-2496)2(2)* (2012)*418-436*

[5]Arora,S.R and Gupta,K., "Paradox in a fractional capacitated transportation problem" , *International Journal Of Research In IT, Management and Engineering (ISSN 2249-1619)* 2(3) (2012), 43-64

[6]Basu, M., Pal, B.B and Kundu, A. "An algorithm for the optimum time cost trade off in a fixed charge bi-criterion transportation problem",*Optimization*,30(1994)53-68

[7]Dahiya, K and Verma, V. "Capacitated transportation problem with bounds on rim conditions", *Europeon journal of Operational Research*, 178 (2007) 718-737

[8]Garfinkel, R.S and Rao, M.R., "The bottleneck transportation problem", *Naval Research Logistics Quarterly*,18(1971) 465-472

[9]Hammer, P.L., "Time minimizing transportation problems", *Naval Research Logistics Quarterly*, 18(1971)487-490

[10]Hirisch ,W.M. and Dantzig ,G.B., " The fixed charge problem", *Naval Research Logistics Quarterly* ,15 (3)(1968) 413-424

[11]Murthy, K.G., "Solving the fixed charge problem by ranking the extreme points", *Operations Research*, 16(1968) 268-279

[12]Pandian , P and Natarajan , G., "A new method for finding an optimal solution for transportation problem", *International Journal of Math. Sci and Engineering Appls (IJMSEA)*,4(2010)59-65

[13]Pandian , P and Natarajan , G., "A new method for solving bottleneck – cost transportation problems", *International Mathematical Forum*, 6(10) (2011)451-460

[14]Sandrock, K., "A simple algorithm for solving small fixed charge transportation problems", *Journal of Operations Research Society* ,39 (5) (1988) 467-475.

[15]Sharma , V., Dahiya , K and Verma , V., "A note on two stage interval time minimization transportation problem", *Australian Society For Operations Research Bulletin* ,27(3) (2008), 12-18.

[16]Sharma,V., Dahiya, K and Verma , V., "A capacitated two stage time minimization transportation problem", *Asia-Pacific Journal of Operations Research* ,27(4)(2010)457-476